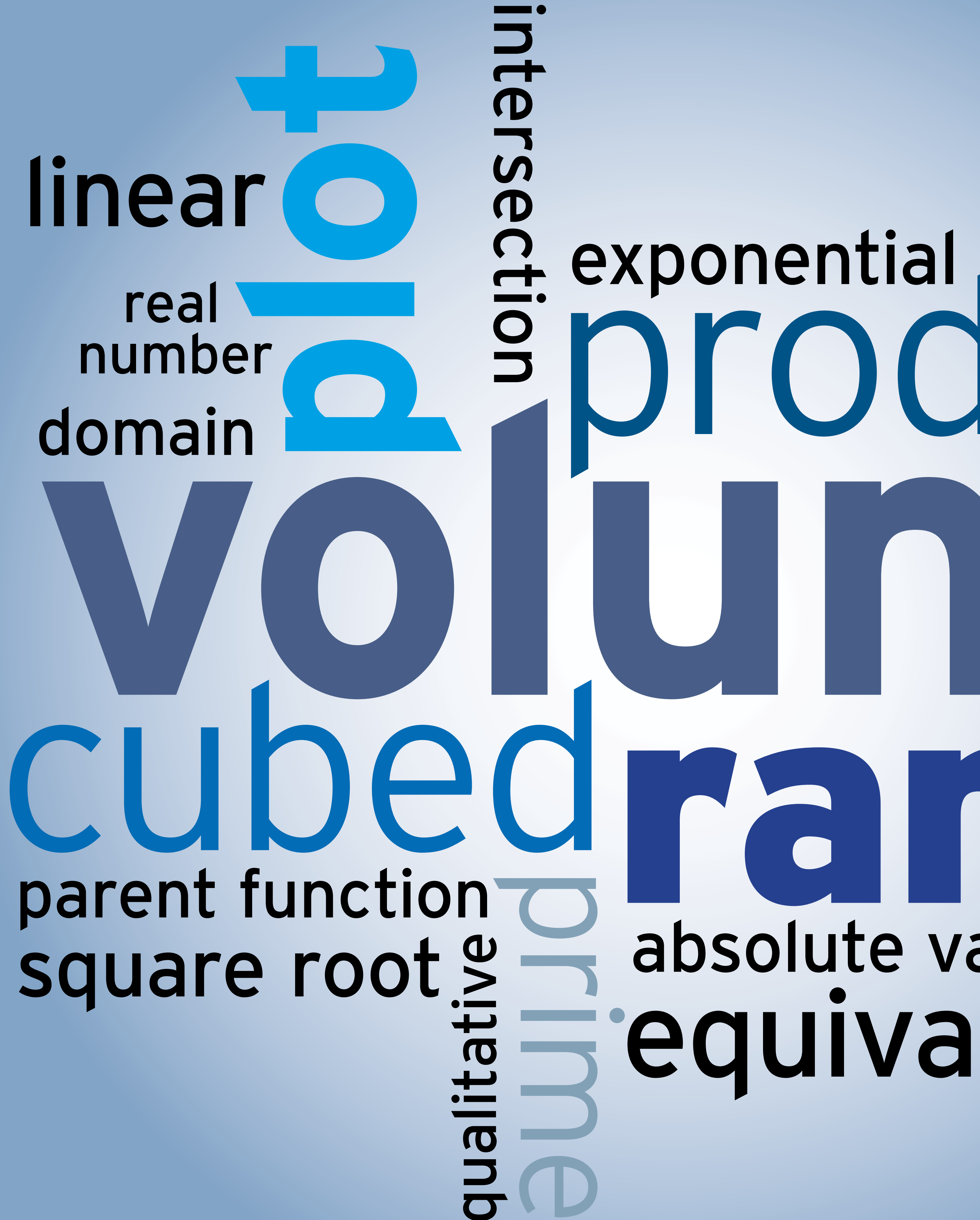
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For ELLs: Vocabulary beyond the Definitions

A classroom teacher discusses ambiguities in mathematics vocabulary and strategies for ELL students in building understanding.

Nancy S. Roberts and Mary P. Truxaw

"But where is the exponent?" Jorge, a tenth-grade English language learner (ELL), asked me while I (co-author Roberts) was talking about the formula for the area of a parallelogram. After much confusion on my part, Jorge said, "Last year you said that the base was the number in a power that was not the exponent. I don't see the exponent, so I don't know where the base is." Aha! I *had* said something like that in algebra class the previous year. However, I had never thought about the two different uses of the word *base* within mathematics.

Although I knew some of the challenges faced by ELL students learning mathematics vocabulary, I had never considered that mathematics, known for its precision, would include ambiguity *within* its vocabulary. In fact, the sixth Standard for Mathematical Practice within the Common Core State Standards for Mathematics (CCSSM) relates to attending to precision: "Mathematically proficient students" need to "communicate precisely to others" and "try to use clear definitions in discussion with others and in their own reasoning" (CCSSI 2010, p. 7).

I thought about Jorge. He had been confident enough and had the language skills to ask for clarification; many ELL students might not. If I had not recognized the connection to my earlier use of mathematics vocabulary, where would this confusion have led? How would I have uncovered it? How would Jorge's confusion have impeded mastering important mathematical practices or communicating precisely? These questions and others led to my investigation into the role of vocabulary development in helping ELL students be successful in mathematics, specifically in first-year algebra.

As I considered the importance of supporting ELL students' mathematics vocabulary, I asked myself a question that would likely arise for many mathematics teachers: "Do I have time to spend on vocabulary development?" Jorge helped me recognize that I had to ask myself a different question: "Can I afford *not* to spend time on vocabulary development?" Many vocabulary strategies that have worked for my students do not add much additional time and enhance not only vocabulary but also the mathematics.

CHALLENGES OF MATHEMATICS VOCABULARY FOR ELLS

Although mathematics language is much more than just learning vocabulary (Moschkovich 1999, 2002; NCTM 2000), vocabulary development is still central to learning to read, write, speak, listen to, and make sense of mathematics (CCSSI 2010; Heinze 2005). I will focus specifically on helping ELL students build better understanding of algebra through vocabulary, sharing outcomes of my own learning about mathematics vocabulary and strategies that worked for my students and me.

Mathematics vocabulary may be more difficult to learn than other academic vocabulary for several reasons.

Table 1 Math Usage vs. Everyday Usage

Vocabulary Word	Mathematics Usage	Everyday Usage
volume	Amount of space	Noise level
product	Result in multiplying	Item produced in manufacturing
plot	To graph a point	A piece of land to build a house
cubed	Raised to the third power	A type of steak or a way to cut vegetables
range	Numerical difference between two values	Stove top
prime	Prime number	Prime rib, prime time

Source: Adams (2003), p. 789

Table 2 Homonyms and Similar Sounding Words

whole – hole two – to – too tenths – tents	eight – ate symbol – cymbal half – have	sum – some sides – size real – reel

Source: Adams (2003), p. 790–91

1. Definitions are filled with technical vocabulary, symbols, and diagrams (Pimm 1987). Teachers need to explicitly help students make sense of this new language (Schlepegrell 2007).
2. Many mathematics concepts can be represented in multiple ways. At least thirteen different terms can mean subtraction (Echevarria, Vogt, and Short 2010; Heinze 2005). Multiplication can be indicated in many ways: "2 times 3," "2 multiplied by 3," and "the product of 2 and 3." To add to the confusion, some words may have similar connotations but vastly different technical meanings—for example, "3 multiplied by 10" and "3 increased by 10" (Heinze 2005).
3. Many mathematics words have multiple meanings. A *quarter* may refer to a coin or a fourth of a whole. Students must learn that the same word in different situations has different meanings, such as asking for a quarter while at a vending machine or while eating a pizza (Moschkovich 2002).
4. The overlap between mathematics vocabulary and everyday English (Kotsopoulos 2007; Moschkovich 2002) is problematic (see **table 1**). The word *product*, for instance, has meaning in everyday English that is completely different from its very specific mathematical meaning.
5. Homonyms and words that sound similar can confuse (Adams 2003). See **table 2** for a partial list.
6. Similarity to native language words can add more confusion. Although these similarities may sometimes be helpful—as when cognates have similar sounds and similar meanings—similarities can also contribute to confusion. For example, the Spanish word for *quarter* is *cuarto*, which can mean "a quarter of an hour"; *quarter* could also mean a room in a house, as in the English usage "your living quarters" (Moschkovich 1999, 2002).

Clearly, vocabulary is an important issue in mathematics classrooms, especially for ELL students.

TEACHING METHODS AND STRATEGIES

A selection of strategies for supporting students' development of mathematics vocabulary and examples of how to use them follow. Suggestions illustrate vocabulary support within an algebra unit but could be adapted for other topics. Two tools that will be highlighted are word walls—organized collections of words displayed in the classroom to support vocabulary development—and graphic organizers—visual charts and representations designed to organize student learning. We will also look at ways in which these tools can encompass vocabulary strategies.

Table 3 Sample Algebra Vocabulary

absolute value	binomial	coefficient	complete the square	conjunction	derive
domain	equivalent	exponential	function	intersection	interval
inverse	linear	monomial	parabola	parent function	piecewise
polynomial	qualitative	quadratic	radicand	range	rational
real number	regression	solution	square root	trinomial	variable

Source: CCSSI (2010), pp. 52–71

Develop a Vocabulary List

Begin by developing a vocabulary list for the unit. **Table 3** shows samples of mathematics vocabulary from the Common Core State Standards for algebra (CCSSI 2010). Along with traditional algebra terms, include vocabulary to support challenges for ELLs, as described earlier (e.g., *symbol* and *whole*). Scaffolding such as word walls and graphic organizers will increase vocabulary usage while reducing cognitive load and stress (Echevarria, Vogt, and Short 2010).

Preteach and Assess

At the start of a unit, it is beneficial to trigger and assess prior knowledge, review previously learned vocabulary, and preteach new vocabulary. Preteaching vocabulary words requires explicit teaching of definitions, pronunciation, and word parts (Paulsen 2007).

Word Walls

One strategy for stimulating and assessing prior knowledge is a word wall. At the beginning of the unit, display all the vocabulary for the unit to act as an *anticipation guide*, a strategy used during preteaching to stimulate interest in a topic and give students a preview of what is to come. One way to use a word wall as a preassessment tool and as the trigger on the first day of a unit is to include a word that does not belong. Then ask small groups to pick out the word and describe why it does not belong. In a graphing unit, for example, the word wall could include the term *scientific notation* along with graphing words such as *slope*, *y-intercept*, *ordered pair*, *xy-intercepts*, and so on. (The nonconforming word would later be removed from the word wall.)

Another way to use word walls for preassessment is to have students organize the words into groups and give reasons for their choices. Words relating to a unit on exponents might be *base*, *exponent*, *denominator*, *numerator*, *polynomial*, *monomial*, *binomial*, *trinomial*, *power*, *reciprocal*, *coefficient*, and *factor*. One student might group *denominator*, *numerator*, and *reciprocal* as words related to fractions; another student might group *base*, *exponent*, and *power* as words describing a power. Listening to discussions provides interactive forms of preassessment. Moreover, student explanations provide opportunities to foster CCSS mathematical practices—for example, communicating precisely to others and constructing viable arguments.

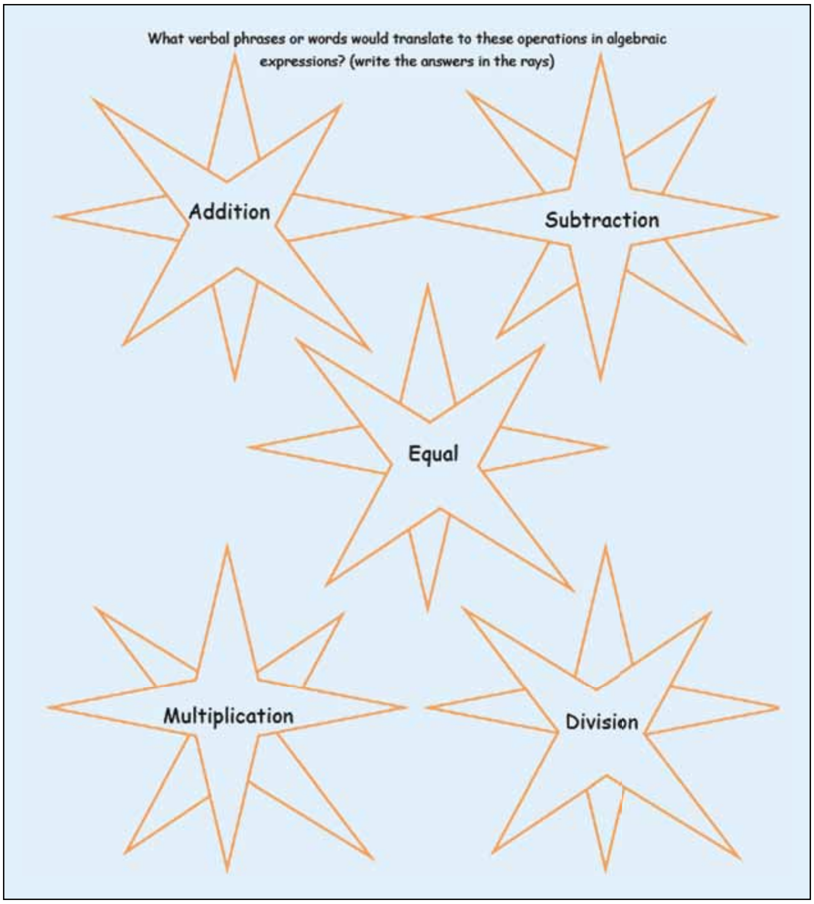

Fig. 1 The points of the stars provide space for students to write phrases that mean the same thing.

Graphic Organizers

Graphic organizers can be useful for activating and assessing students' prior knowledge, organizing different ways to express basic mathematical concepts, and organizing vocabulary for long-term retention. One organizer includes eight-sided stars with words for arithmetic operations and equality (see **fig. 1**). Working with partners, students list words that could be used for each operation. Then they add to their lists by comparing these in small groups. Finally, the class as a whole reviews the words. This class review is a time to make connections to the mathematical concepts, to address misconceptions, and to include words and phrases that are often confusing—for instance, "4 less than x" to mean "x minus 4."

Teach and Reteach

Researchers have provided many suggestions for explicitly teaching and reteaching vocabulary (see, e.g., Adams 2003; Gee 1996; Moschkovich 2002; Paulsen 2007). The focus here will be on word walls and graphic organizers.

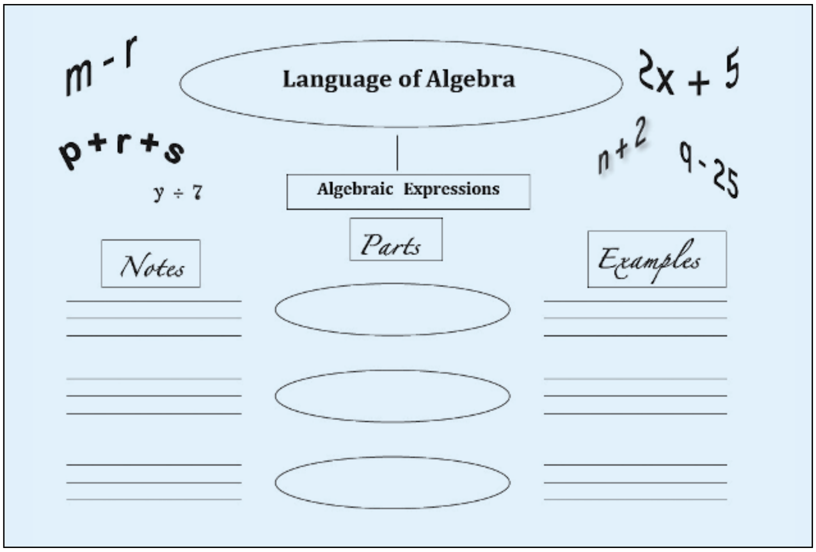

Fig. 2 *Variable*, *constant*, and *operation* would be appropriate entries in the ovals in the middle column.

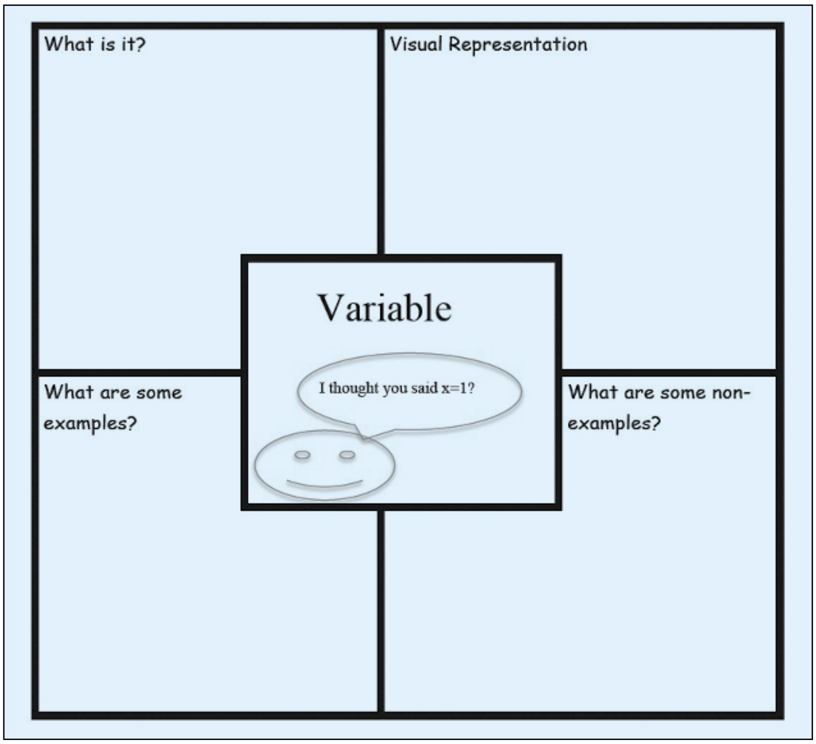

Fig. 3 A Frayer model is useful for some vocabulary.

Word Walls

Word walls are also useful within instructional units. A key idea is that word walls should be interactive, not static. After explicitly teaching words in the context of the unit, add definitions, examples, and diagrams to the words on the wall. Using nonexamples can help refine or clarify definitions (Adams 2003). In addition, real-life situations can provide context for algebra vocabulary and concepts (Paulsen 2007).

A helpful strategy is to start with informal definitions (while preteaching and assessing prior knowledge) and then transition to formal definitions (NCTM 2000). For example, the informal definition "a variable is a letter" may lead to "a variable is a symbol that represents a number" and finally to "a variable is a symbol, usually a letter, that is a quantity that can have different values." Informal definitions help students construct their own meaning, but formal definitions help them understand and apply concepts presented in mathematics textbooks (Adams 2003).

Ongoing interactive use of the word wall helps students see its value. As the year progresses, students use the word wall when answering verbal questions, when writing responses to essential questions on tests, and at other times when vocabulary usage is emphasized.

Graphic Organizers

Graphic organizers are beneficial within a unit of study to build and reinforce mathematics language. A graphic organizer entitled The Language of Algebra provides an opportunity to teach or reteach the parts of an algebraic expression by giving definitions and examples in the context of expressions (see **fig. 2**). In this specific organizer, the "parts" section (middle column) could list *variable*, *constant*, and *operation*, with notes and examples for each in the left and right columns. Similar language organizers could be developed for other topics.

A Frayer model is a specific graphic organizer that is useful when vocabulary terms are confusing or closely related (Barton and Heidema 2002). The model contains four sections: definition, facts, examples, and nonexamples (see **fig. 3** for an example related to the term *variable*). Both research (Adams 2003; Paulsen 2007) and personal experience demonstrate that nonexamples can be particularly powerful in helping refine and clarify definitions. When students ask, "How about this?" or "How about that?" they can refer to the example and nonexample sections. New misunderstandings that are uncovered can be added to the “nonexample” section. Sometimes substituting sections to suit the situation can be useful—for instance, using essential characteristics and nonessential characteristics or symbolic representation and graphical representation as sections. Students frequently refer to their organizers during lessons or when reviewing for tests.

Provide Repetition and Support Long-Term Retention

All students benefit from repeated exposure to vocabulary; however, ELLs require more repetition to integrate vocabulary into their mathematical understanding. In addition, students may need assistance in organizing their vocabulary knowledge into long-term memory (Adams 2003). Using vocabulary words within context while referring to the definitions (Echevarria, Vogt, and Short 2010) can be helpful. Providing different examples or diagrams each time the word is used helps avoid confusion and brings depth to students’ growing understanding.

Word Walls

Reinforcing vocabulary from the interactive word wall can support long-term retention. A simple idea is to take four to five minutes at the end of class to play password or charades, using words from current or previous word walls. Another idea is to encourage and facilitate instructional conversations (Cazden 2001) that can support long-term retention of mathematics language and build meaning about mathematical concepts (NCTM 2000). Word walls can scaffold these conversations. When small groups discuss a mathematics problem, points can be awarded for appropriate use of words from the word wall—for example, using words such as *formula*, *variables*, *equations*, *graphs*, and *order of operations* when discussing using algebra in the real world.

Graphic Organizers

The graphic organizers used throughout a unit can and should be revisited to support long-term retention. In addition, new graphic organizers can be introduced to review previously learned vocabulary and concepts. For example, an organizer with a formal definition, specific properties or special cases, and some examples could be used to review the concept of factors (see **fig. 4**).

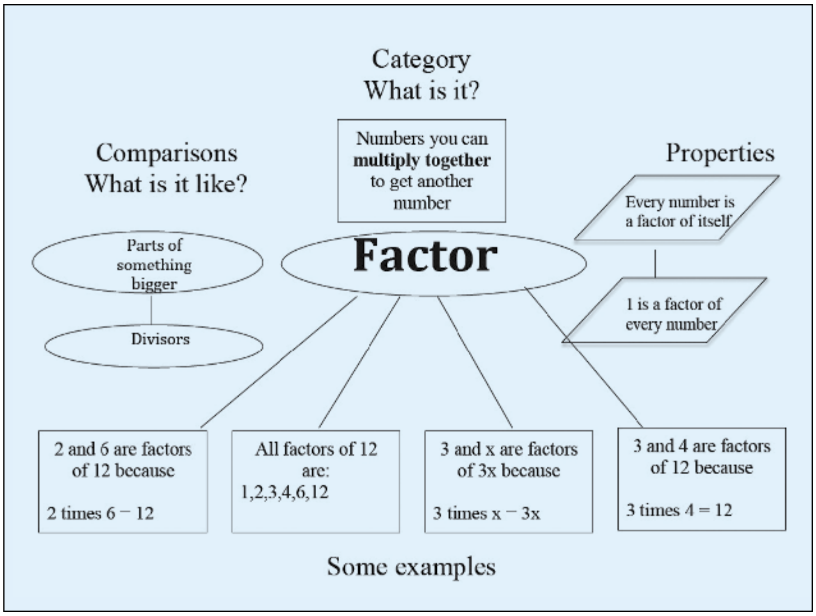

Fig. 4 This graphic organizer would be useful during review of the concept of factors.

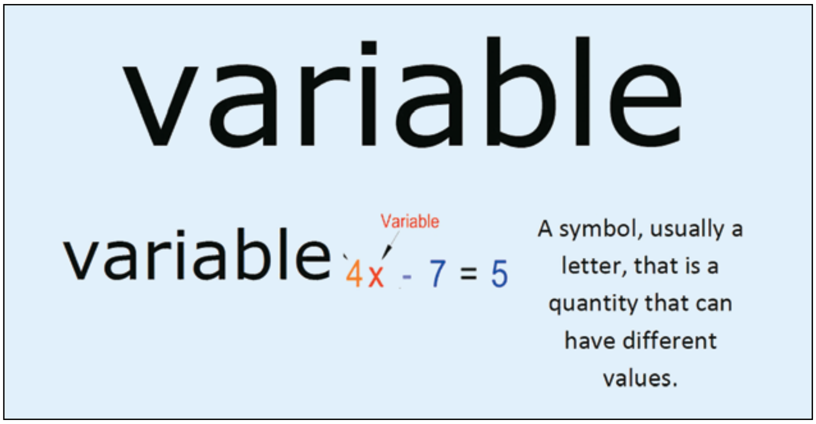

Fig. 5 This word wall entry can be folded so that only the vocabulary word is showing.

TEACHER AWARENESS

Along with reading research literature, mathematics teachers should build their own understanding of the challenges that their ELL students face. Awareness of the confusion caused by symbols and diagrams, concepts that can be represented with multiple terms, words that have multiple meanings, and the overlap between mathematics vocabulary and everyday usage can help teachers provide appropriate emphasis or explicit teaching.

HELPFUL HINTS

Word Walls

A simple way to make a word wall is to use a hanging pocket schedule organizer (typically used by elementary school teachers). After deciding on the unit vocabulary list (see **table 3**), type the words into a document (in landscape mode), with one word on each line. Center each word and enlarge it so that it fills a line of the paper. On the next line, type the word, its definition, a diagram, and an example. After printing, fold the paper so that the word is on one side and the expanded definition is on the other (see **fig. 5**). Slide the pieces into the organizer with the words showing. As the unit progresses and the words are discussed in context, reverse the paper so that the expanded definition is revealed.

Graphic Organizers
Many Internet sites—for example, CAST (www.cast.org) and Thinkport (www.thinkport.org)—have sample graphic organizers that can be used as is or customized. Teachers need not limit themselves to mathematics organizers; many excellent vocabulary organizers, such as Frayer models, come from other content areas. Providing a graphic organizer can help connect content within the unit and then can be used later as a review. Colored paper can assist with organization. In my class, colored paper means "keep it forever." Color makes important graphic organizers easy to find (I can say, "Pull out the red graphic organizer on variables"). At the end of the year, unit organizers make a good, concise way to review.

REFLECTIONS AND RECOMMENDATIONS

As I reflect on my experiences and those of my students, I am reminded of Jorge's confusion about mathematics vocabulary. His question has led me to increase my own awareness of the challenges related to mathematics vocabulary that ELL students face and strategies that I might use to support these students.

To help ELL students develop essential mathematical practices (CCSSI 2010), I recommend the use of word walls and graphic organizers to support vocabulary development. Specifically, I recommend the following:

- Select vocabulary words for a unit and post these on the day that the unit is introduced.
- Assess students' current understanding.
- Refer to the words throughout the unit, adding to the definitions and giving context.
- Provide frequent opportunities for students' misunderstanding to come to light.
- Use graphic organizers to help clarify the meaning of words and support long-term retention of vocabulary.

In addition to using word walls and graphic organizers, teachers should continue to investigate ideas available through books, journal articles, and websites (there are lots of good ideas out there). And, of course, listen to your students—that's the first step in supporting them.

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1

Why and How to Differentiate Math Instruction

STUDENTS IN ANY CLASSROOM differ in many ways, only some of which the teacher can reasonably attend to in developing instructional plans. Some differences will be cognitive—for example, what previous concepts and skills students can call upon. Some differences will be more about learning style and preferences, e.g., whether the student learns better through auditory, visual, or kinesthetic approaches. Other differences will be more about preferences, including behaviors such as persistence or inquisitiveness or the lack thereof and personal interests.

THE CHALLENGE IN MATH CLASSROOMS

Although teachers in other subject areas sometimes allow students to work on alternative projects, it is much less likely that teachers vary the material they ask their students to work with in mathematics. The math teacher will more frequently teach all students based on a fairly narrow curriculum goal presented in a textbook. The teacher will recognize that some students need additional help and will provide as much support as possible to those students while the other students are working independently. Perhaps this occurs because **differentiating instruction** in mathematics is a relatively new idea. Perhaps it is because teachers may never have been trained to really understand how students differ mathematically. However, students in the same math classroom clearly do differ mathematically in significant ways. Teachers want to be successful in their instruction of all students. Understanding differences and differentiating instruction are important processes for achievement of that goal.

The National Council of Teachers of Mathematics (NCTM), the professional organization whose mission it is to promote, articulate, and support the best possible teaching and learning in mathematics, recognizes the need for differentiation. The first principle of the NCTM *Principles and Standards for School Mathematics* reads, "Excellence in mathematics education requires equity—high expectations and strong support for all students" (NCTM, 2000, p. 12).

In particular, NCTM recognizes the need for accommodating differences among students, taking into account both their readiness and their level of mathematical talent/interest/confidence, to ensure that each student can learn important mathematics. "Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students" (NCTM, 2000, p. 12).

THE PARTICULAR CHALLENGE IN GRADES 6–12

The challenge for teachers of grades 6–12 is even greater than in the earlier grades, particularly in situations where students are not streamed. Although there is much evidence of the value, particularly for the struggling student, of being in heterogeneous classrooms, the teacher in those rooms must deal with significant student differences in mathematical level. While some students are still struggling with their multiplication facts or addition and subtraction with decimals, others are comfortable with complex reasoning and problem solving involving fractions, decimals, and percents. The differences between students' mathematical levels, beginning as far back as kindergarten or grade 1, continue to be an issue teachers must face all through the grades.

Where some see the answer as streaming, many believe that the answer is a differentiated instruction environment in a destreamed classroom.

WHAT IT MEANS TO MEET STUDENT NEEDS

One approach to meeting each student's needs is to provide tasks within each student's **zone of proximal development** and to ensure that each student in the class has the opportunity to make a meaningful contribution to the class community of learners. Zone of proximal development is a term used to describe the "distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, p. 86).

Instruction within the zone of proximal development allows students, whether with guidance from the teacher or by working with other students, to access new ideas that are beyond what the students know but within their reach. Teachers are not using educational time wisely if they either are teaching beyond the student's zone of proximal development or are providing instruction on material the student already can handle independently. Although other students in the classroom may be progressing, the student operating outside his or her zone of proximal development is often not benefiting from the instruction.

For example, a teacher might be planning a lesson on calculating the whole when a percent that is greater than 100% of the whole is known, using a problem such as asking students to determine what number 30 is 210% of. Although the skill that the teacher might emphasize is solving a proportion such as

$$\frac{210}{100} = \frac{30}{x}$$

the more fundamental objective is getting students to recognize that solving a percent problem is always about determining a ratio equivalent to one where the second term is 100.

Although the planned lesson is likely to depend on the facts that students can work algebraically with two fractions, one involving a variable, and that they understand the concept of a percent greater than 100%, a teacher could effectively teach a meaningful lesson on what percent is all about even to students who do not have those abilities. The teacher could allow the less developed students to explore the idea of determining equivalent ratios to solve problems using percents less than 100% with ratio tables or other more informal strategies (rather than formal proportions) while the more advanced students are using percents greater than 100% and more formal methods. Only when the teacher felt that the use of percents greater than 100% and algebraic techniques were in an individual student's zone of proximal development would the teacher ask that student to work with those sorts of values and strategies. Thus, by making this adjustment, the teacher differentiates the task to locate it within each student's zone of proximal development.

ASSESSING STUDENTS' NEEDS

For a teacher to teach to a student's zone of proximal development, first the teacher must determine what that zone is. This can be accomplished by using prior assessment information in conjunction with a teacher's own analysis to ascertain a student's mathematical developmental level. For example, to determine an 8th-grade student's developmental level in working with percents, a teacher might use a diagnostic to find out whether the student interprets percents as ratios with a second term of 100, relates percents to equivalent fractions and/or decimals, can represent a percent up to 100% visually, can explain what 150% means, and recognizes that solving a percent problem involves determining an equivalent ratio.

Some tools to accomplish this sort of evaluation are tied to developmental continua that have been established to describe students' mathematical growth (Small, 2005a, 2005b, 2006, 2007, 2010). Teachers might also use locally or personally developed tools to learn about students' prior knowledge. Only after a teacher has determined a student's level of mathematical sophistication, can he or she meaningfully address that student's needs.

PRINCIPLES AND APPROACHES TO DIFFERENTIATING INSTRUCTION

Differentiating instruction is not a new idea, but the issue has been gaining an ever higher profile for mathematics teachers in recent years. More and more, educational systems and parents are expecting the teacher to be aware of what each individual student—whether a struggling student, an average student, or a gifted student—needs and to plan instruction to take those needs into account. In the past, this was less the case in mathematics than in other subject areas, but now the expectation is common in mathematics as well.

There is general agreement that to effectively differentiate instruction, the following elements are needed:

- ***Big Ideas.*** The focus of instruction must be on the **big ideas** being taught to ensure that they are addressed, no matter at what level (Small, 2009a; Small & Lin, 2010).
- ***Prior Assessment.*** Prior assessment is essential to determine what needs different students have (Gregory & Chapman, 2007; Murray & Jorgensen, 2007).
- ***Choice.*** There must be some aspect of choice for the student, whether in content, process, or product.

Teaching to Big Ideas

The Curriculum Principle of the NCTM *Principles and Standards for School Mathematics* states that "A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades" (NCTM, 2000, p. 14).

Curriculum coherence requires a focus on interconnections, or big ideas. Big ideas represent fundamental principles; they are the ideas that link the specifics. For example, the notion that benchmark numbers are a way to make sense of other numbers is equally useful for the 6th-grader who is trying to place –22 on a number line, the 8th-grader who relates π to the number 3.14, or the 10th-grader who is trying to estimate the sine of a 50° angle. If students in a classroom differ in their readiness, it is usually in terms of the specifics and not the big ideas. Although some students in a classroom where estimating the value of radicals is being taught may not be ready for that precise topic, they could still deal with the concept of estimating and why it is useful in simpler situations.

Big ideas can form a framework for thinking about "important mathematics" and supporting standards-driven instruction. Big ideas cut across grade bands. There may be differences in the complexity of their application, but the big ideas remain constant. Many teachers believe that curriculum requirements limit them to fairly narrow learning goals and feel that they must focus instruction on meeting those specific student outcomes. Differentiation requires a different approach, one that is facilitated by teaching to the big ideas. It is impossible to differentiate too narrow an idea, but it is always possible to differentiate instruction focused on a bigger idea.

Prior Assessment

To determine the instructional direction, a teacher needs to know how students in the classroom vary in their mathematical developmental level. This requires collecting data either formally or informally to determine what abilities or what deficiencies students have. Although many teachers feel they lack the time or tools to undertake prior assessment on a regular basis, the data derived from prior assessment should drive how instruction is differentiated.

Despite the importance of prior assessment, employing a highly structured approach or a standardized tool for conducting the assessment is not mandatory. Depending on the topic, a teacher might use a combination of written and oral questions and tasks to determine an appropriate starting point for each student or to determine what next steps the student requires. An example of one situation is described below, along with steps teachers might take, given the student responses.

Consider the task below:

> The Beep-Beep pager company charges $30 to set up a client with a pager along with a $7.50 monthly fee. The Don't Miss Them pager company charges $9 a month, but no set up fee. How long would you have to own a pager before the Beep-Beep deal is a better one?

Students might respond to the task in very different ways. Here are some examples:

- Joshua immediately raises his hand and just waits for the teacher to help him.
- Blossom says that 30 + 7.50 + 9 = 46.50, so it would take 46.50 months.
- Madison writes: $y = 30 + 7.50x$ and $y = 9x$, so $9x = 30 + 7.5x$. That means $1.5x = 30$, and $x = 30 \div 1.5$. Since $x = 20$, it would be 20 months.
- Lamont starts a table like this one, but forgets to add the terms to answer the question.

Beep-Beep	Don't Miss Them
37.50	9
7.50	9
7.50	9
7.50	9

- Hannah uses an appropriate table, and extends it until the Beep-Beep value is less and counts the number of entries.

Beep-Beep	Don't Miss Them
37.50	9
45	18
52.50	27
60	36
. . .	. . .
180	180
187.50	189

- Latoya reasons that the difference in price is $1.50 a month, so you just divide 30 by 1.50 to figure out how many months it would take to make up the extra cost at the start. She calculates the value to be 20 and then indicates that after 21 months (assuming whole numbers of months), the Beep-Beep plan is better.

The teacher needs to respond differently based on what has been learned about the students. For example, the teacher might wish to:

- Encourage Joshua to be more independent or set out an alternate related problem that is more suitable to his developmental level
- Help Blossom understand that just because there are three numbers in a problem, you don't automatically add, and emphasize the importance of reading carefully what the problem requests
- Encourage Madison's thoughtful approach to the problem, but help her see that she still hasn't really answered the question posed
- Ask Lamont to label his columns and tell what each represents, then ask him how the table might help him solve the problem
- Ask Hannah to think of a way she could have used her idea without having to show every single row in the table
- Ask Latoya, who clearly is thinking in a very sophisticated way, to create a different scenario where the Beep-Beep plan would not be better until, for example, 31 months

By knowing where the students are in their cognitive and mathematical development, the teacher is better able to get a feel for what groups of students might need in the way of learning and can set up a situation that challenges each individual while still encouraging each one to take risks and responsibility for learning (Karp & Howell, 2004).

Choice

Few math teachers are comfortable with the notion of student choice except in the rarest of circumstances. They worry that students will not make "appropriate" choices.

However, some teachers who are uncomfortable differentiating instruction in terms of the main lesson goal are willing to provide some choice in follow-up activities students use to practice the ideas they have been taught. Some of the strategies that have been suggested for differentiating practice include the use of menus from which students choose from an array of tasks, tiered lessons in which teachers teach to the whole group and vary the follow-up for different students, learning stations where different students attempt different tasks, or other approaches that allow for student choice, usually in pursuit of the same basic overall lesson goal (Tomlinson, 1999; Westphal, 2007).

For example, a teacher might present a lesson on using the exponent laws to simplify expressions to all students, and then vary the follow-up. Some students might work only with simple situations; these tasks are likely to involve simple multiplications of pairs of numbers with the same base, such as $2^5 \times 2^7$. Other students might be asked to work with situations where a variety of laws might be called on at once, such as simplifying $2^5 \times (2^7)^2 \div 2^5$. Some students might deal with even more challenging questions, such as determining two factors for 1 million where neither one involves a power of 10 (e.g., $10^6 = 2^6 \times 5^6$). By using prior assessment data, the teacher is in a better position to provide appropriate choices.

TWO CORE STRATEGIES FOR DIFFERENTIATING MATHEMATICS INSTRUCTION: OPEN QUESTIONS AND PARALLEL TASKS

It is not realistic for a teacher to try to create 30 different instructional paths for 30 students, or even 6 different paths for 6 groups of students. Because this is the perceived alternative to one-size-fits-all teaching, instruction in mathematics is often not differentiated. To differentiate instruction efficiently, teachers need manageable strategies that meet the needs of most of their students at the same time. Through the use of just two core strategies, teachers can effectively differentiate instruction to suit all students. These two core strategies are the central feature of this book:

- **Open questions**
- **Parallel tasks**

Open Questions

The ultimate goal of differentiation is to meet the needs of the varied students in a classroom during instruction. This becomes manageable if the teacher can create a single question or task that is inclusive not only in allowing for different students to approach it by using different processes or strategies but also in allowing for students at different stages of mathematical development to benefit and grow from attention to the task. In other words, the task is in the appropriate zone of proximal development for the entire class. In this way, each student becomes part of the larger learning conversation, an important and valued member of the learning community. Struggling students are less likely to be the passive learners they so often are (Lovin, Kyger, & Allsopp, 2004).

A question is open when it is framed in such a way that a variety of responses or approaches are possible. Consider, for example, these two questions, each of which might be asked of a whole class, and think about how the responses to each question would differ:

> ***Question 1:*** Write the quadratic $y = 3x^2 - 12x + 17$ in vertex form.
>
> ***Question 2:*** Draw a graph of $y = 3x^2 - 12x + 17$. Tell what you notice.

Question 1 is a fairly closed question. If the student does not know what vertex form is, there is no chance he or she will answer Question 1 correctly. In the case of Question 2, a much more open question, students simply create the graph and notice whatever it is that they happen to notice—whether that is the vertex, that the shape is parabolic, that it opens upward, and so on.

Strategies for Creating Open Questions. This book illustrates a variety of styles of open questions. Some common strategies that can be used to construct open questions are described below:

- Turning around a question
- Asking for similarities and differences
- Replacing a number, shape, measurement unit, and so forth with a blank
- Asking for a number sentence

Turning Around a Question. For the turn-around strategy, instead of giving the question, the teacher gives the answer and asks for the question. For example:

What is 75% of 20?	→	15 is a percent of a number. What percent of what number is it?
What is the hypotenuse of a right triangle if the legs are 3 units and 4 units long?	→	One side of a right triangle is 5 units long. What could the other side lengths be?

Asking for Similarities and Differences. The teacher chooses two items—two numbers, two shapes, two graphs, two probabilities, two measurements, and so forth—and asks students how they are alike and how they are different. Inevitably, there will be many good answers. For example, the teacher could ask how the number $\sqrt{2}$ is like the number $\sqrt{5}$ and how it is different. A student might realize that both are irrational numbers, both are less than 3, both are greater than 1, and both are side lengths of squares with a whole number of units of area.

Replacing a Number with a Blank. Open questions can be created by replacing a number (or numbers) with a blank and allowing the students to choose the number(s) to use. For example, instead of asking for the surface area of a cone with radius 4" and height 15", the teacher could ask students to choose numbers for the radius and height and then determine the surface area. By allowing choice, the question clearly can go in many directions. Most importantly, students can choose values in such a way that their ability to demonstrate understanding of the concept being learned is not compromised by extraneous factors such as the complexity of the calculations required of them.

Asking for a Sentence. Students can be asked to create a sentence that includes certain words and numbers. For example, a teacher could ask students to create a sentence that includes the number 0.5 along with the words "sine," "rational," and "amplitude," or a sentence that includes the words "linear" and "increasing" as well as the numbers 4 and 9. The variety of sentences students come up with will often surprise teachers. For example, for the second situation, a student might produce any of the sentences below and many more:

- *An increasing linear pattern could include the numbers 4 and 9.*
- *In a linear pattern starting at 4 and increasing by 9, the tenth number will be 85.*
- *A linear pattern that is increasing by 9 grows faster than one that is increasing by 4.*

Shortcut for Creating Open Questions. A teacher can sometimes create an open question by beginning with a question already available, such as a question from a text resource. Here are a few examples:

Graph and solve this linear system of equations: $0.5x + 0.6y = 5.4$ $-x + y = 9$	→	Write two equations involving both x and y. Determine values for x and y that make both of them true.
Solve for m: $\frac{4m}{5} - \frac{1}{2} = \frac{-25}{2}$	→	The solution to an equation is $m = -15$. The equation involves a fraction. What might the equation be?
Matthew has 20 ounces of a 40% salt solution. How much salt should he add to make it a 45% solution?	→	Matthew has 20 ounces of a 40% salt solution. He wants a solution with a greater percentage of salt. Decide on the percentage of salt you want. Tell how much salt to add.

What to Avoid in an Open Question. An open question should be mathematically meaningful. There is nothing wrong with an occasional question such as *What do you like about algebra?* but questions that are focused more directly on big ideas or on curricular goals are likely to accomplish more in terms of helping students progress satisfactorily in math.

Open questions need just the right amount of ambiguity. They may seem vague, and that may initially bother students, but the vagueness is critical to ensuring that the question is broad enough to meet the needs of all students.

On the other hand, teachers must be careful about making questions so vague that they deter thinking. Compare, for example, a question like *What is infinity?* with a question like *How do you know that there are an infinite number of decimals* *between 0 and 1?* In the first case, a student does not know whether what is desired is a definition for the word, something philosophical, or the symbol ∞. The student will most likely be uncomfortable proceeding without further direction. In the second case, there is still ambiguity. Some students may wonder if a particular approach is desired, but many students will be comfortable proceeding by using their own strategies.

The reason for a little ambiguity is to allow for the differentiation that is the goal in the use of open questions. Any question that is too specific may target a narrow level of understanding and not allow students who are not at that level to engage with the question and experience success.

A Different Kind of Classroom Conversation. Not only will the mathematical conversation be richer when open questions are used, but almost any student will be able to find something appropriate to contribute.

The important point to notice is that the teacher can put the same question to the entire class, but the question is designed to allow for differentiation of response based on each student's understanding. All students can participate fully and gain from the discussion in the classroom learning community.

This approach differs, in an important way, from asking a question, observing students who do not understand, and then asking a simpler question to which they can respond. By using the open question, students gain confidence; they can answer the teacher's question right from the start. Psychologically, this is a much more positive situation.

Multiple Benefits. There is another benefit to open questions. Many students and many adults view mathematics as a difficult, unwelcoming subject because they see it as black and white. Unlike subjects where students are asked their opinions or where they might be encouraged to express different points of views, math is viewed as a subject where either you get it or you don't. This view of mathematics inhibits many students from even trying. Once they falter, they lose confidence and assume they will continue to falter—they may simply shut down.

It is the responsibility of teachers to help students see that mathematics is multifaceted. Any mathematical concept can be considered from a variety of perspectives, and those multiple perspectives actually enrich its study. Open questions provide the opportunity to demonstrate this.

Fostering Effective Follow-Up Discussion. Follow-up discussions play a significant role in cementing learning and building confidence in students. Thus, it is important for teachers to employ strategies that will optimize the effectiveness of follow-up discussions to benefit students at all developmental levels.

To build success for all students, it is important to make sure that those who are more likely to have simple answers are called on first. By doing so, the teacher will increase the chances that these students' answers have not been "used up" by the time they are called on.

CTD Professional Development Lesson Plan Feedback Form

Evaluation Date ____

Teacher Name ____ Lesson plan #: 1 2 3

School and District ____

CCSS Standard used:

A. Coherence:

B. Progression: (Does the lesson plan reflect the 4 steps of formative assessment: Clarify intended learning, Elicit evidence, Interpret evidence, and Act on evidence?)

C. Use of Assessment:

D. Additional comments

Core Time Digital Professional Development Lesson Plan

Lesson Title: Equivalent Ratios defined through the value of a ratio
Lesson #: 1

CCSS Standard used: <u>6RP.1, 6RP.2</u>

Lesson Objectives: <u>Build on the ratio skills to include writing ratios written as fractions (value), and find equivalent ratios written in the simplest form in real world and mathematical problems.</u>

LESSON CONTENT

Introduction

Declarative knowledge – Describe the content of the lesson
<u>Equivalent ratios have the same value</u>

Conditional knowledge—Describe why students learn content and the conditions for using it
<u>The target and rubric are introduced. Ratios are useful in many everyday situations. Sometimes a recipe, sometimes an exercise program, sometimes—and most important to them right now—time spent at something and the grade or improved skill (reading, skateboarding, guitar…).</u>

Development

Procedural knowledge—Describe the steps for guiding students to acquire the strategy/content
<u>This lesson is built on prior lessons of ratios. There is a review of ratios and settings and the two ways to write a ratio that have been introduced up to this point. Then it is introduced as a fraction form. It begins with a picture representation. It is then written as a simplified fraction, as a reinforcement, for equivalent ratios. The fraction is referred to as a value. This skill is then practiced in a variety contexts.</u>
<u>Each of these steps is introduced by me, practiced by them and checked by them, practiced by them and checked by a buddy, then practiced by them and checked by me (problem set). There is an exit slip and two homework problems.</u>

Differentiated instruction—<u>Describe strategies for engaging ELL/SPED students</u>
<u>Students who need it are given one-on-one, or small group, by the paraprofessional, some are doing a smaller assignment, some advanced are encouraged to continue on their own finding ratios throughout the period or day that they notice and write their own.</u>

Formative Assessment—Describe instruments for monitoring learning and forms of intervention
Formative assessment continues throughout the lesson from the introduction through the exit slip. Children's explanations often lead to a moment when clarification is required. A child's written response, coloring response, and/or the ability to move through the skill as the gradual release of responsibility requires, occur throughout the lesson and may require modification.

Closure

Final Assessment—Describe relevant instruments for monitoring and evaluating students learning
Final assessments are done throughout the unit of study on ratios, rate, proportion and percentage. There are graded assignments done on a weekly basis as well as an end of the unit basis. Today, there has been adequate practice for an interim independent, teacher created classroom assessment for skills covered up to this point.