# Make sense of problems and persevere in solving them

***When presented with a problem, I can make a plan, carry out my plan, and evaluate its success.***

## BEFORE...

**EXPLAIN** the problem to myself.

- *Have I solved a problem like this before?*

**ORGANIZE** information...

- *What is the question I need to answer?*
- *What is given?*
- *What is not given?*
- *What are the relationships between known and unknown quantities?*
- *What tools will I use?*
- *What prior knowledge do I have to help me?*

## DURING...

**PERSEVERE**

**MONITOR** my work

**CHANGE** my plan if it isn't working out

**ASK** myself, "Does this make sense?"

## AFTER...

**CHECK**

- *Is my answer correct?*
- *How do my representations connect to my algorithms?*

**EVALUATE**

- *What worked?*
- *What didn't work?*
- *What other strategies were used?*
- *How was my solution similar to or different from my classmates'?*

# Reason abstractly and quantitatively

***I can use reasoning habits to help me contextualize and decontexualize problems.***

## CONTEXTUALIZE

I can take numbers and put them in a real-world context.

For example, if given
3 x 2.5 = 7.5
I can create a context:

*I walked 2.5 miles per day for 3 days. I walked a total of 7.5 miles.*

## DECONTEXTUALIZE

I can take numbers out of context and work mathematically with them.

For example, if given
'I walked 2.5 miles per day for 3 days. How far did I walk?',
I can write and solve

3 x 2.5 = 7.5

Reasoning Habits include 1) *creating an understandable representation of the problem solved*, 2) *considering the units involved*, 3) *attending to the meaning of quantities*, and 4) *using properties to help solve problems*.

# Construct viable arguments and critique the reasoning of others

***I can make conjectures and critique the mathematical thinking of others.***

I can ***construct***, ***justify***, and ***communicate*** arguments by...

- considering context
- using examples and non-examples
- using objects, drawings, diagrams and actions

I can ***critique the reasoning of others*** by...

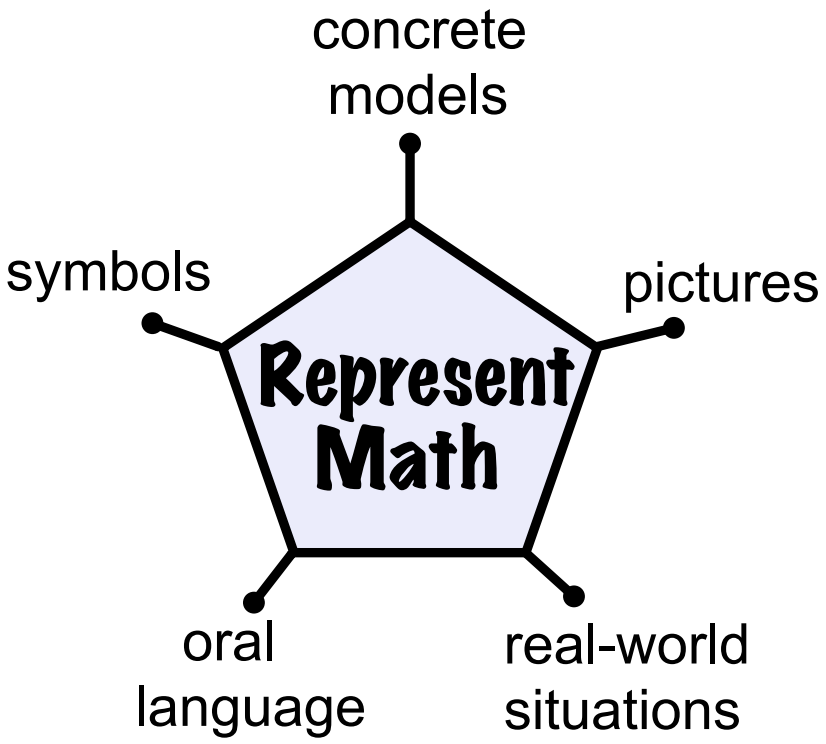
- listening
- comparing arguments
- identifying flawed logic
- asking questions to *clarify* or *improve arguments*

# Model with mathematics

***I can recognize math in everyday life and use math I know to solve everyday problems.***

## I can...

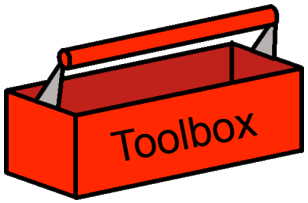
- make assumptions and estimate to make complex problems easier
- identify important quantities and use tools to show their relationships
- evaluate my answer and make changes if needed



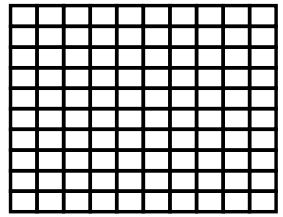
# Use appropriate tools strategically

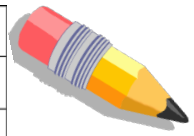
***I know when to use certain tools to help me explore and deepen my math understanding.***

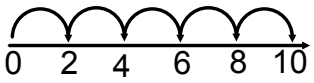
I have a math toolbox.

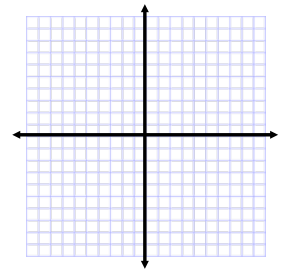


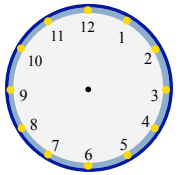

- I know *HOW* to use math tools.
- I know *WHEN* to use math tools.
- I can reason: *"Did the tool I used give me an answer that makes sense?"*









$a \times b = b \times a$

$V = b \times h$

# Attend to precision

***I can use precision when solving problems and communicating my ideas.***

## Problem Solving

- I can calculate *accurately*.
- I can calculate *efficiently*.
- My answer matches what the problem asked me to do – *estimate* or find an *exact answer*.

## Communicating

- I can SPEAK, READ, WRITE, and LISTEN mathematically.
- I can correctly use...
  - math symbols
  - math vocabulary
  - units of measure

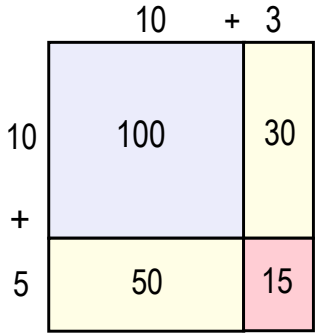
# Look for and make use of structure

***I can see and understand how numbers and spaces are organized and put together as parts and wholes.***

## Numbers

For Example:

- Base 10 structure
- operations and properties
- terms, coefficients, exponents




13 x 15

(10 + 3) x (10 + 5)

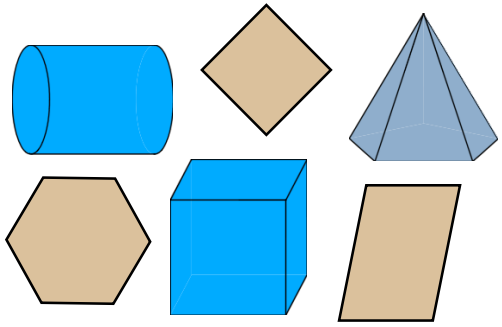
100 + 30 + 50 + 15

195

## Spaces

For Example:

- dimension
- location
- attributes
- transformation



# Look for and express regularity in repeated reasoning

***I can notice when calculations are repeated. Then, I can find more efficient methods and short cuts.***

For example: 25 ÷ 11

$$
\begin{array}{r}
2.2727 \\
11\overline{)25.0000} \\
-22 \\
\hline
30 \\
-22 \\
\hline
80 \\
-77 \\
\hline
30 \\
-22 \\
\hline
80 \\
-77 \\
\hline
30
\end{array}
$$

I am repeating this calculation. The quotient is a repeating decimal.

# Make sense of problems and persevere in solving them

Make meaning of a problem and look for entry points to its solution

Analyze givens, constraints, relationships, and goals

Make conjectures about the meaning of the solution

Develop a plan

Monitor and evaluate progress and change course if necessary

Check answers to problems and determine if the answer makes sense

# Reason abstractly and quantitatively

Make sense of quantities and their relationships

Represent symbolically (ie: Equations, expressions)

Manipulate equations (attends to the meaning of the quantities, not just computes them)

Understands and uses different properties and operations

# Construct viable arguments and critique the reasoning of others

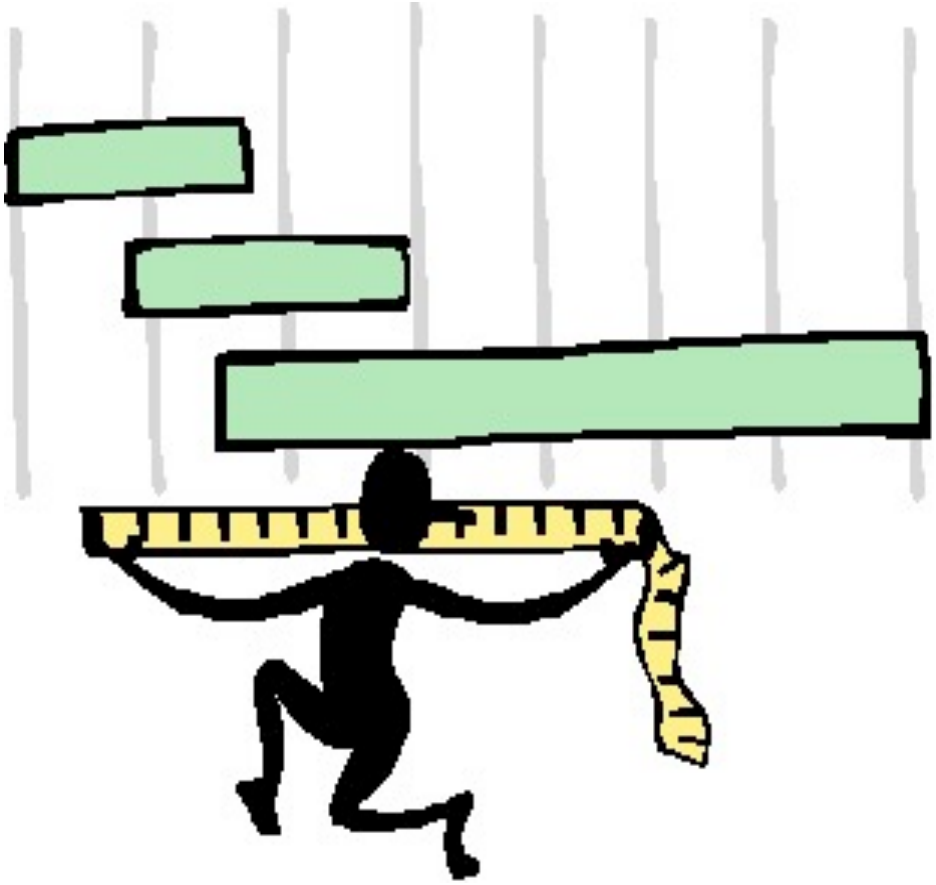

and written communication

Understand and use definitions in previously established results when justifying results

Attempts to prove or disprove conjectures through examples and counterexamples

Communicates and defends their mathematical reasoning using objects, drawings, diagrams, actions, verbal

# Model with mathematics



Solve math problems arising in everyday life

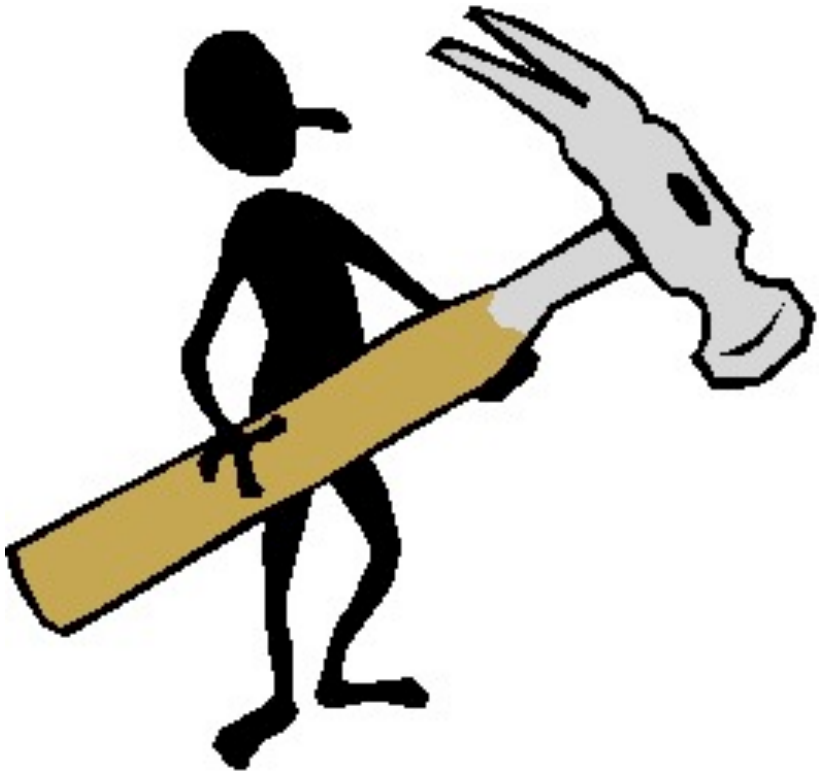
Apply assumptions and approximations to simplify complicated tasks

Use tools such as diagrams, two-way tables, graphs, flowcharts and formulas to simplify tasks

Analyze relationships mathematically to draw conclusions

Interpret results to determine whether they make sense

# Use appropriate tools strategically



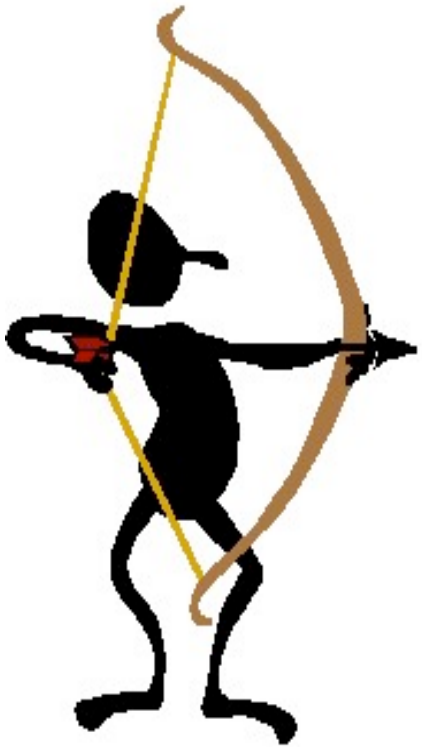
Decide which tools will be most helpful (ie: ruler, calculator, protractor)

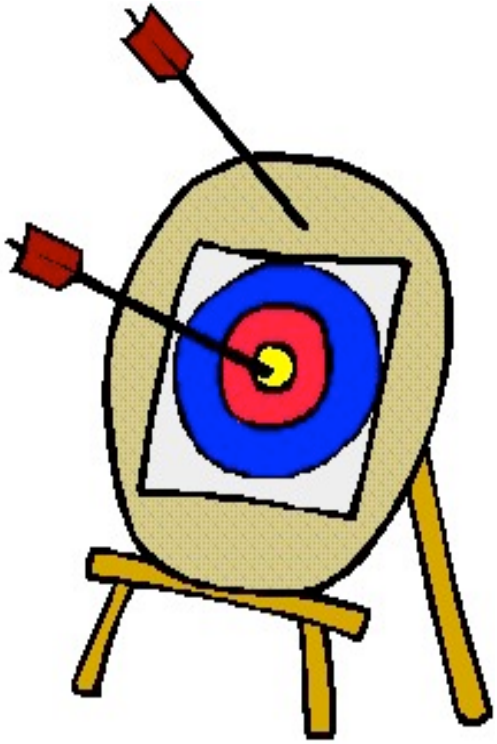
Detect possible errors by strategically using estimation and other mathematical knowledge

Make models that enable visualization of the results and compare predictions with data

Use technological tools to explore and deepen understanding of concepts

# Attend to precision





Communicate precisely to others

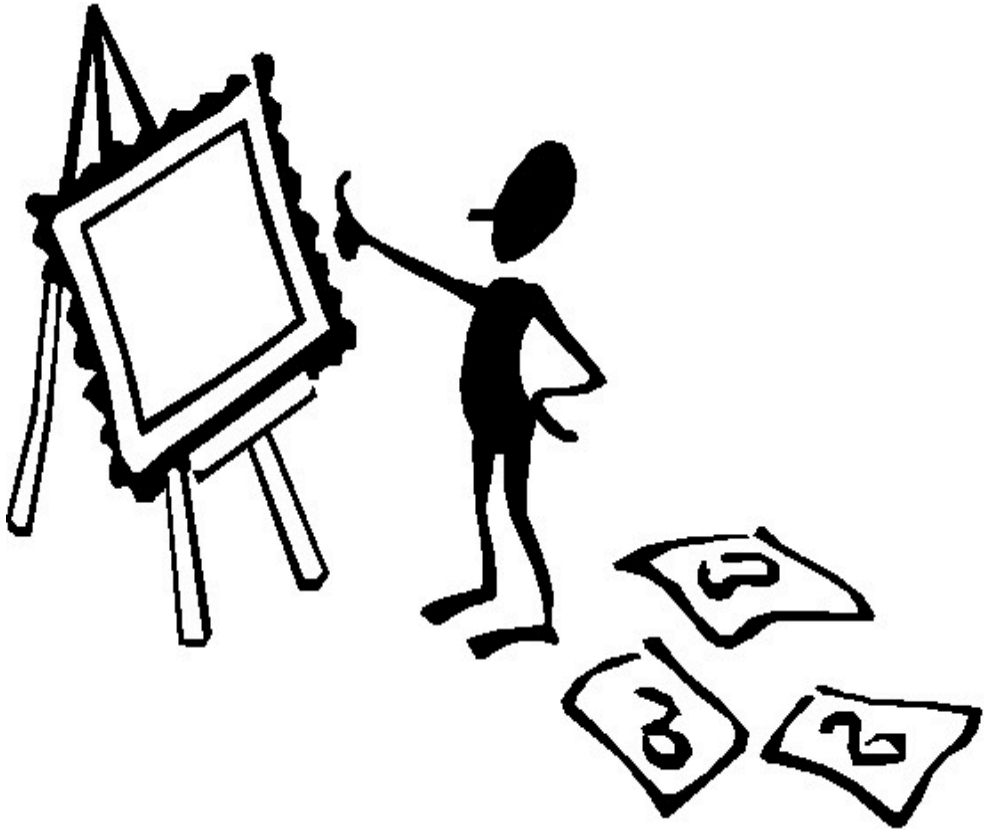
Use clear definitions in discussion with others

State the meaning of the symbols consistently and appropriately

Calculate accurately and efficiently

Accurately label axes and measures in a problem

# Look for and make use of structure

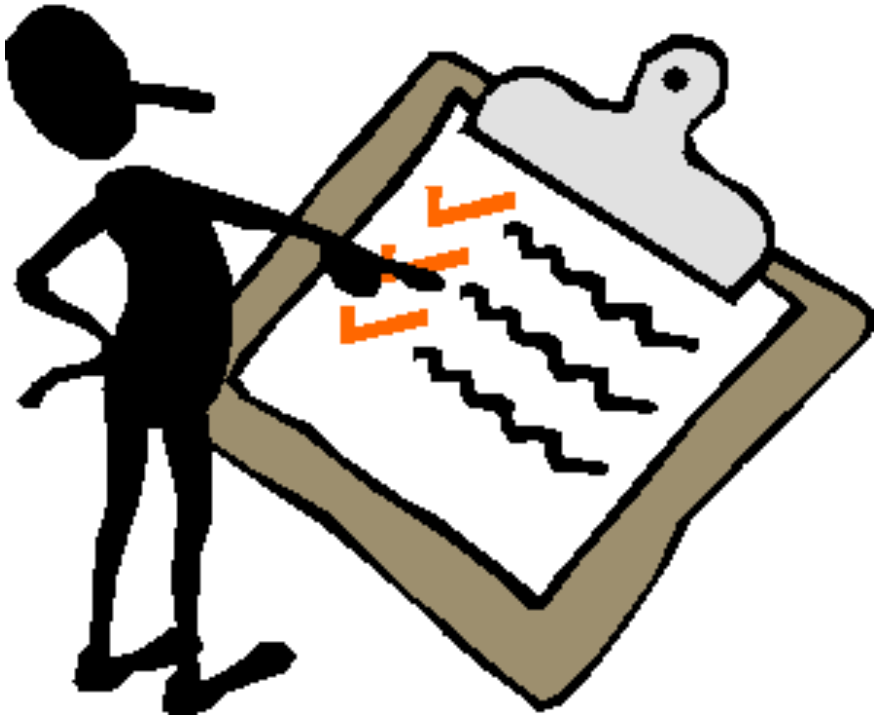


Look closely to determine a pattern or structure

Step back for an overview and shift perspective

See complicated things a being composed of single objects or several smaller objects

# Look for and express regularity in repeated reasoning



Identify calculations that repeat

Look both for general methods and for shortcuts

Maintain oversight of the process, while attending to the details

Continually evaluate the reasonableness of results

# Formative Assessment of the Mathematical Practices

Author: Morning Star Williams
Owner: Morning Star Williams

### Subjects and Domains

- Math - Practice - 1. Make sense of problems and persevere in solving them
- Math - Practice - 2. Reason abstractly and quantitatively
- Math - Practice - 3. Construct viable arguments and critique the reasoning of others
- Math - Practice - 4. Model with mathematics
- Math - Practice - 5. Use appropriate tools strategically
- Math - Practice - 6. Attend to precision
- Math - Practice - 7. Look for and make use of structure
- Math - Practice - 8. Look for and express regularity in repeated reasoning
- Not Subject Specific

### Grades

- 6 - Sixth Grade
- 7 - Seventh Grade
- 8 - Eighth Grade
- 9 - Ninth Grade
- 10 - Tenth Grade
- 11 - Eleventh Grade
- 12 - Twelfth Grade

### Intended End Users

- Student
- Teacher

### Intended Student Populations

- All Students

## Summary

The resource Formative Assessment of the Mathematical Practices highlights implementation of the CCSS Mathematical Practices as a formative assessment tool in the classroom. Students clarify intended learning around the CCSS Mathematical Practices: using the Mathematical Practices Graphic Organizer, by watching a video, and creating their own criteria. Using the Mathematical Practices Student Rubric, students collect evidence on their proficiency for each of the eight mathematical practices and cite evidence from their learning. Students then analyze their own evidence and act on that evidence to set goals for how they will improve their own mathematical practice.

### Attributes of the Formative Assessment Process

- Clarify Intended Learning
- Elicit Evidence
- Interpret Evidence
- Act on Evidence

## Specific Connection to the Formative Assessment Process

Students use the Mathematical Practices Graphic Organizer to clarify intended learning targets. Also, students use Mathematical Practices Student Rubric to clarify intended learning targets and set success criteria around the CCSS Mathematical Practices. Finally, with clarity of intended learning around the CCSS Mathematical Practices, students elicit, interpret and act on their own evidence to improve their own learning.

## Student Engagement to the Formative Assessment Process

Students are actively engaged in the formative assessment process by using metacognitive strategies to increase their level of proficiency with respect to the eight mathematical practices. The CCSSMP Mathematical Practices Student Rubric call attention to this engagement in the formative assessment process.

**Media Types**

- Video
- Document

**Educational Use**

- Activity
- Assessment
- Classifying
- Cooperative Learning
- Creative Response
- Differentiation
- Discovery Learning
- Discussion/Debate
- Experiential
- Interactive
- Nonlinguistic
- Presentation
- Questioning
- Reflection
- Technology
- Writing

**Technologies Required for use in Classroom**

- Internet
- Tablets/Computers for Teacher
- LCD Projector

**Geographics Settings**

- Urban
- Suburban
- Rural

**License For Primary Material**

- Smarter Balanced Terms of Service (owned by the contributor)

Students are given an opportunity to clarify intended learning by creating their own working definition of what the Mathematical Practices are and are not. Students are engaged in eliciting and interpreting their own evidence with respect the CCSSMP during learning. Lastly, students act on evidence by reflecting on their own engagement with the Mathematical Practices, and then setting goals for future learning opportunities.

## Specific Connection to the Common Core State Standards

The instructional resource is focused on the formative assessment process and the CCSS Mathematical Practices. The resource does not have a direct connection to specific CCSS Mathematic Content Standards it could be used in conjunction with a wide variety of CCSSM Content Standards. The resource addresses the CCSS Mathematical Practices.

## Learning Goals

The students will use the Mathematical Practices Graphic Organizer (MPGO) to clarify intended learning of the Common Core State Standards for Mathematical Practices (CCSSMP). The students will use the Mathematical Practices Student Rubric (MPSR) to set success criteria, collect and analyze evidence, and reflect on their learning processes and proficiencies. The students will self assess and set personal goals related to the CCSSMPs.

## Success Criteria

The student can articulate a working definition of the CCSSMPs. The student can identify, assess, and set goals for engaging the CCSSMPs. The student can implement one or more goals to increase their proficiencies in CCSSMPs.

## Context(s) in Which the Resource Could Be Used

Formative Assessment of the Mathematical Practices has been used in numerous High school math classroom settings, but can be adapted for Middle/Intermediate/Junior High school math classroom. The resource calls for students to work in groups, as well as independently, during the formative assessment process.

## Supporting Evidence

The resource has been used in numerous Title I classroom settings. The use of metacognitive strategies with students as a formative assessment tool has proven to be impactful in a variety of classroom settings.

## Principles, Literature, or Research

The resource is representative of research and learning theories that assert the importance of Formative Instructional processes, metacognitive, and learning strategies. Also, the resource material Mathematical Practices Student Rubric is adapted from Standards of Student Practice in Mathematics Proficiency Matrix (Hull, Balka, and Harbin Miles, 2011), which is based upon previous research and reasoning from the National Council of Teachers of Mathematics (NCTM) in the Principles and Standards for School Mathematics (2000) and from the National Research Council (NRC) in the book, “Adding It Up” (2001). The resource is informed by widely accepted practices in Mathematical Education.

# Link to External Materials

*from*

The Teaching Channel

| | |
|---|---|
| Title of the Material(s) | *Owning the Common Core Math Practices* |
| Link to External Material(s) | *https://www.teachingchannel.org/videos/owning-the-common-core* |

# *Formative Assessment of the Mathematical Practices*

***Formative Assessment of the Mathematical Practices*** is an Instructional Resource for teachers and students. The resource addresses all four aspects of the Formative Assessment process: clarify intended learning, elicit evidence, interpret evidence, and act on evidence. Students use the Mathematical Practices Student Rubric to clarify intended learning targets around the mathematical practices. Students will clarify intended learning around the CCSS Mathematical Practices, elicit, interpret and act on their own evidence. Students are actively engaged in the formative assessment process by using metacognitive strategies to increase their level of proficiency with respect to the eight mathematical practices. Students clarify intended learning around the CCSS Mathematical Practices: using the Mathematical Practices Graphic Organizer, by watching a video, and creating their own criteria. Using the Mathematical Practices Student Rubric, students collect evidence on their proficiency for each of the eight mathematical practices and cite evidence from their learning. Students then analyze their own evidence and act on that evidence to set goals for how they will improve their own mathematical practice.

- Student Learning targets
  - The students will use the ***Mathematical Practices Graphic Organizer (MPGO)*** to clarify intended learning of the Common Core State Standards for Mathematical Practice (CCSSMP).
  - The students will use the ***Mathematical Practices Student Rubric (MPSR)*** to set success criteria, collect and analyze evidence, and reflect on their learning processes and proficiencies.
  - The students will self assess and set personal goals related to the CCSSMPs.

- Success Criteria
  - The student can articulate a working definition of the CCSSMPs.
  - The student can identify, assess, and set goals for engaging the CCSSMPs.
  - The student can implement one or more goals to increase their proficiencies in CCSSMPs.

- Materials needed: Internet, LCD Projector, Computer for Teacher, one copy of ***MPGO*** per group or pair, one copy of CCSSMP for each student, and one copy of ***MPSR*** for each student.

## Clarify intended learning

- Review the Eight CCSSMP.
  - Prepare reference materials, CCSSMP, (http://www.corestandards.org/Math/Practice/), One copy of CCSSMP for each student.
  - Prepare MPGO one copy of ***MPGO*** per group or pair. In pairs or groups, have students use the ***MPGO*** to deconstruct one of the eight CCSSMP and create a working definition of each practice. Chart-Post-Share out to the group.
- Watch the video from the teachingchannel.org video library https://www.teachingchannel.org/videos/owning-the-common-core
- Have students create a list of "look for's" - what students would consider evidence of each CCSSMP.
- Prepare one copy of ***MPSR*** for each student. Review the accompanying ***MPSR*** with students and cite any explicit connections between what students consider "look for's" and the ***MPSR.*** Teacher can provide more examples or answer questions that arise while reviewing the rubric. Note: the MPSR was adapted from the ***Standards of Student Practice in Mathematics Proficiency Matrix*** (Hull, Balka, and Harbin Miles, 2011).

## Elicit Evidence

- When applicable, have students categorize which of the eight CCSSMP were addressed during the lesson.
- Once students are fluent in accurately categorizing evidence of each CCSSMP, have students self-assess their level of proficiency using the ***MPSR.***
- Differentiation – reflect on their work with one CCSSMP through a focused lens (i.e. While you do this activity today, focus on MP #3a Constructing Viable Arguments… Using the ***MPSR***, how would you rate your level of proficiency on MP #3a? What evidence do you have to support your self-assessment of MP#3a?).

## Interpret Evidence

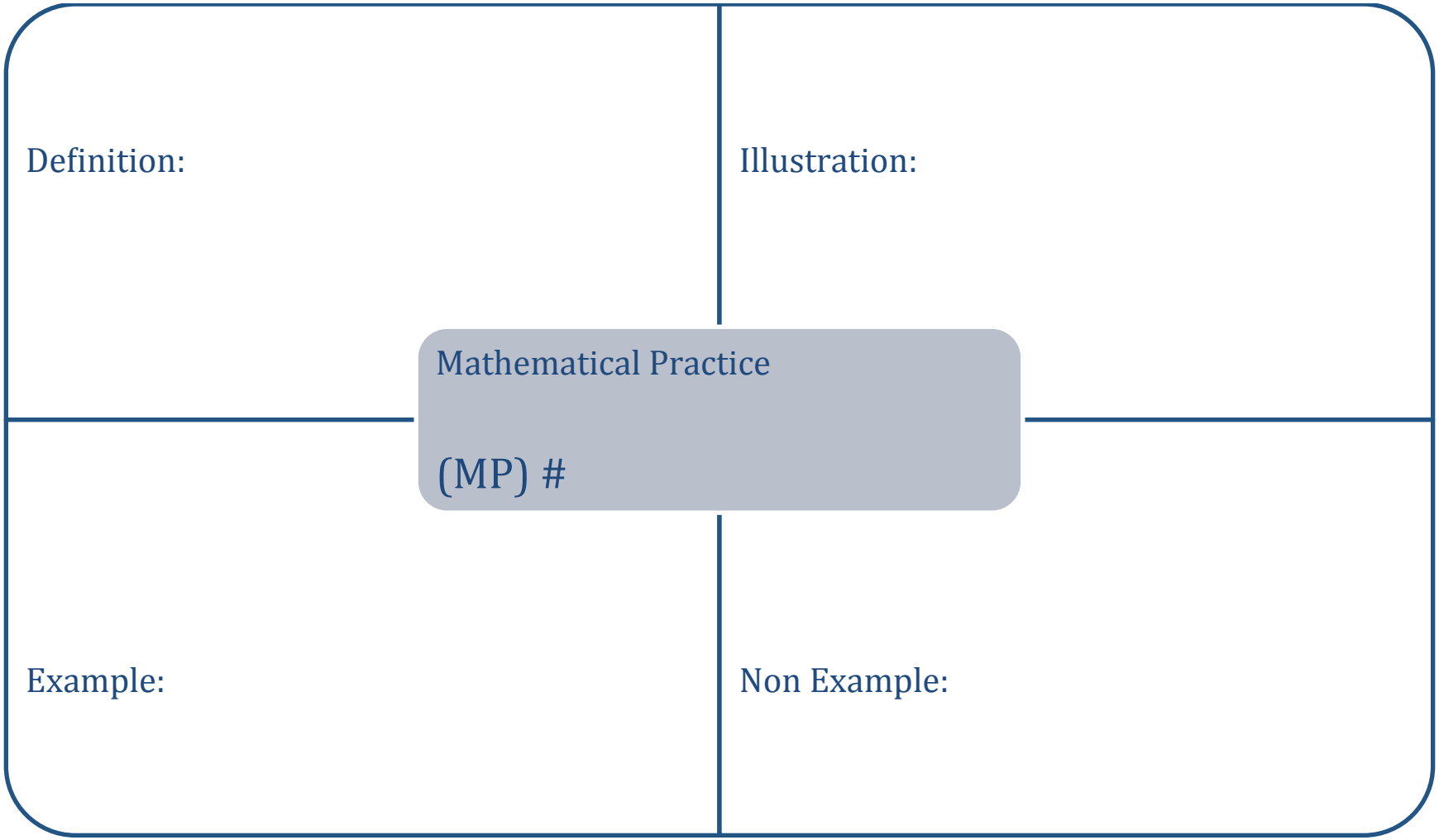
- Over the course of several lessons or a unit of study, have students formally reflect on their gains in proficiency of each CCSSMP.
- Have students self-assess them selves using the ***MPSR*** and cite specific evidence from their own work/learning that supports their level of proficiency rating.
  - Student: I rate myself an Intermediate "IN" on MP3a, because I can use accurate vocabulary in my justifications and explanations of my own thinking and the thinking of others. However, I'm not Advanced "A", because I can't always explain why a solution is correct. I'm an Intermediate "IN" as demonstrated by my work evidence..., …, and ….

- Have students identify one or more CCSSMP that they are strong/weak in
  - Student: I'm strong in CCSSMP #3a Constructing Viable Arguments, because I can use accurate vocabulary in my justifications and explanations of my own thinking and the thinking of others.
  - Student: I'm weak in CCSSMP #1b Persevere in solving them, because I often quit a difficult problem when I don't get it right the first time.

## Act on Evidence

- Have students set goals for one or more CCSSMP that they are strong/weak in
  - Student: I will try many different solution paths to a challenging problem, and only ask for help if I get stuck.
- Allow time for student presentations of their CCSSMPs growth to peers.
- A teacher may collect and tabulate the proficiencies of each CCSSMP to provide Formative Assessment data on trends of students and classes in relation to the CSSSMPs.
- A teacher may reflect and identify a focus to guide instructional next steps, and provide additional opportunities for student(s) around specific CCSSMP(s).

Mathematical Practices Graphic Organizer



# Mathematical Practices Student Rubric

| CCSSMP | *Students: I can* | *(I)=Initial* | *(IN)=Intermediate* | *(A)=Advanced* |
|---|---|---|---|---|
| 1a | **Making Sense of problems** | Explain my thought processes in solving a problem one way. | Explain my thought processes in solving a problem and representing it in several ways. | Discuss, explain, and demonstrate solving a problem with multiple representations and in multiple ways. |
| 1b | **Persevere in solving them** | Stay with a challenging problem for more than one attempt. | Try several approaches in finding a solution, and only seek hints if stuck. | Struggle with various attempts over time, and learn from previous solution attempts. |
| 2 | **Reason abstractly and quantitatively** | Reason with models or pictorial representations to solve problems. | Are able to translate situations into symbols for solving problems. | Convert situations into symbols to appropriately solve problems as well as convert symbols into meaningful situations. |
| 3a | **Construct viable arguments** | Explain my thinking for the solution I found. | Explain my own thinking and thinking of others with accurate vocabulary. | Justify and explain, with accurate language and vocabulary, why a solution is correct. |
| 3b | **Critique the reasoning of others** | Understand and discuss other ideas and approaches. | Explain other students' solutions and identify strengths and weaknesses of the solution. | Compare and contrast various solution strategies and explain the reasoning of others. |
| 4 | **Model with Mathematics** | Use models to represent and solve a problem, and translate the solution to mathematical symbols. | Use models and symbols to represent and solve a problem, and accurately explain the solution representation. | Use a variety of models, symbolic representations, and technology tools to demonstrate a solution to a problem. |
| 5 | **Use appropriate tools strategically** | Use the appropriate tool to find a solution. | Select from a variety of tools the ones that can be used to solve a problem, and explain their reasoning for the selection. | Combine various tools, including technology, explore and solve a problem as well as justify their tool selection and problem solution. |
| 6 | **Attend to precision** | Communicate my reasoning and solution to others. | Incorporate appropriate vocabulary and symbols when communicating with others. | Use appropriate symbols, vocabulary, and labeling to effectively communicate and exchange ideas. |
| 7 | **Look for and make use of structure** | Look for structure within mathematics to help me solve problems efficiently. | Compose and decompose number situations and relationships through observed patterns in order to simplify solutions. | See complex and complicated mathematical expressions as component parts. |
| 8 | **Look for and express regularity in repeated reasoning** | Look for obvious patterns, and use if/then reasoning strategies for obvious patterns. | Find and explain subtle patterns. | Discover deep, underlying relationships, i.e. uncover a model or equation that unifies the various aspects of a problem such as discovering an underlying function. |