Instructional Notes

These notes include suggestions and other information relevant to the Task. They also include ideas about delivering the instruction and engaging students in their learning.

The CCSS Addressed

- Understand congruence in terms of rigid motions
    - CCSS.MATH.CONTENT.HSG.CO.B.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
    - CCSS.MATH.CONTENT.HSG.CO.B.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
    - CCSS.MATH.CONTENT.HSG.CO.B.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
- Apply geometric concepts in modeling situations (applies to Act on Evidence)
    - CCSS.MATH.CONTENT.HSG.MG.A.3 Apply geometric methods to solve design problems (e.g., congruence and rigid motions to design an object or structure to satisfy physical constraints).
- Mathematical Practices
    - CCSS.MATH.PRACTICE.MP3 Construct viable arguments and critique reasoning of others.
    - CCSS.MATH.PRACTICE.MP6 Attend to precision.

Instructional Challenge

- In grades 9–12, students begin to formalize their geometry experiences from earlier grades. During this time, the concept of congruence is explored from the perspective of rigid motions or transformations (e.g., translations, rotations, and reflections). Students need to understand the measure preserving properties of these motions and how such motions can be used to define congruence. They need opportunities to explore the principle of superposition and to establish triangle congruence criteria using rigid motions.

Task Context

In this task, your students will explore congruent triangles and the ways they can use rigid motions to prove that two triangles are congruent.

Group size: Students should work in groups of 2-4 to maximize task potential.

Differentiation: Some students need manipulatives to be able to visualize the intended learning while other students grasp it better through verbalization. Other students, such as ELLs and SWDs may require both. This clarification activity will focus the students on the intended learning.

Congruent figures are important in our practical world. In small groups, students brainstorm where they might find congruent figures in our practical world. Instruct students to think about everyday objects.

1. Give students time to brainstorm while walking around listening for different types of congruent figures.
2. If the students get stumped, then encourage them to consider congruent figures they might find in the classroom (i.e., ceiling and floor tiles, walls, tables, desks, chairs, windows, drawers, cupboards, etc.).

3. Then challenge them to identify where they might find congruent figures outside of the classroom (i.e., homes, buildings, roads, amusement rides, water slides, food containers, dishes, silverware, etc.).
4. Have each group choose a different object to share out.
5. Show presentation of different congruent shapes: wallpaper patterns, quilts, bathroom tiles, kaleidoscopes, Ferris wheels, automobile manufacturing plants, stained glass windows, sidewalks, bridges, or playground equipment.

Because of the regular occurrence of congruent figures in the real world, it is important to know a lot about such figures. In this task, students explore congruent triangles and the ways rigid motions can be used to prove that two triangles are congruent.

Students work on computers to use Geogebra to explore congruent figures and rigid motion.

Ask students: Previously, you have learned about rigid motion and transformations of rigid motions. What are some rigid motions that you have learned about? Response: translations, rotations, and reflections. What is particularly interesting about rigid motions or what property do each of these share? Response: preserve size and shape.

Part 1

In Part 1, students explore how they can use rigid motions to prove that two triangles are congruent.

Students begin with the pair of triangles in the dynamic geometry software file provided along with a worksheet. This worksheet has the following instructions and questions:

- In the dynamic geometry software file provided, you will find two triangles that are labeled such that two pairs of corresponding sides are congruent and one pair of corresponding angles is congruent.

- Use a sequence of rigid motions to prove that the two triangles shown are congruent.
  Note: You must record the steps that you take to show the two triangles are congruent. Along with this record, include a reason or reasons why the images formed after each step are congruent to the original triangle.

- Compare and contrast your work with your peers. Are their steps the same as yours? If not, are their steps valid? If so, can you find a different sequence of steps that illustrate congruence?

As students work on the task, you should be looking for their comfort level with the ideas of translations, reflections, and rotations. The students need to use rigid motions to not only transform figures and predict the effect they have on a specific figure, but also to determine when two figures are congruent. As you listen to their discussions, you are listening for students using the definitions of rigid motion and congruence as well as the measure preserving properties of rigid motion.

Question Matrix

Use the following question matrix to help you plan the questions you will ask students as they work on the task:

Elicit Evidence	Interpret Evidence	Act On Evidence
1. What vocabulary are the students using to describe the rigid motion?	1. Are students using the vocabulary of rigid motion: translations, reflections, and rotations? • Yes, they are using the vocabulary of rigid motion: translations, reflections, and rotations. • No, they are not using the vocabulary of rigid motion: translations, reflections, and rotations.	1. Review the rigid motion vocabulary with the students. Provide a word bank to the small group or post these on the board for students' reference.
2. What meaning of the vocabulary of rigid motion are the students using?	2. Are students using the meaning of the vocabulary words for rigid motion correctly? • Yes, they are using the vocabulary of rigid motion: translations, reflections, and rotations. • No, they are not using the vocabulary of rigid motion: translations, reflections, and rotations.	2. Put key words that describe the rigid motion vocabulary on a sticky note to the small group or on the whiteboard for the large group to reference (Rotation = turn, Translation = slide, and Reflection = flip).
3. Can the students use the rigid motions to transform figures?	3. Are the students able to translate, reflect, and rotate parts of one triangle to another? • Yes, they are able to translate, reflect, and rotate parts of one triangle to another. • No, they are not able to translate, reflect, and rotate parts of one triangle to another.	3. Guide the students to translate a point. Suggest they may try to translate a segment, then reflect, and then rotate.
4. Can the students use the rigid motions to predict the effect they have on a specific figure?	4. Are the students able to predict the effect translation, reflection, and rotation has on a specific figure? • Yes, they are able to predict the effect translation, reflection, and rotation has on a specific figure. • No, they are not able to predict the effect translation, reflection, and rotation has on a specific figure.	4. Guide the students to predict a point. Suggest they may try to predict by translating a segment, then reflecting, and then rotating.
5. Can the student determine whether the two figures are congruent?	5. Are the students able to understand that rigid motions of shapes preserve the shape and size of the shape? • Yes, they are able to understand that rigid motions of shapes preserve the shape and size of the shape. • No, they are not able to understand that rigid motions of shapes preserve the shape and size of the shape.	5. Have the students compare the shape and size before the rigid motion. Once they understand that rigid motions preserve the properties of rigid motion, ask them to explain at what point they were convinced the triangles were congruent and why.

Universal Design for Learning Principles

Multiple Means of Representation
- Students can make drawings or use a computer program to represent their solutions.
- Students are offered manipulatives to support their understanding.

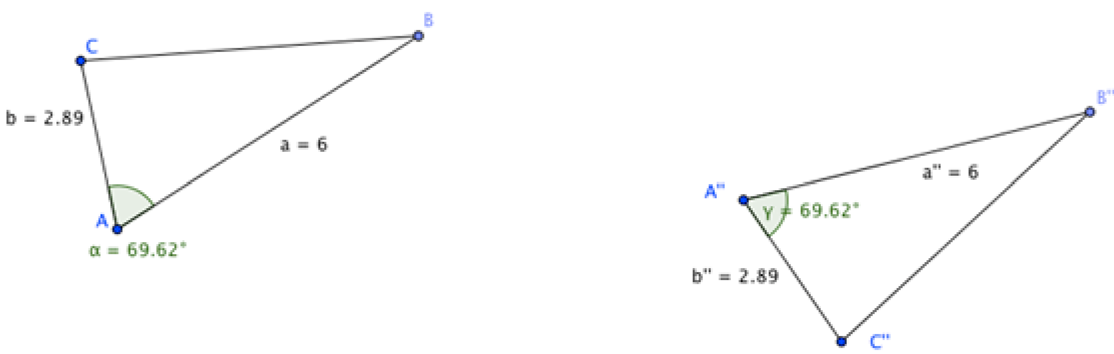
Multiple Means of Demonstration
- Students demonstrate their knowledge through participating in oral discourse, using manipulatives, making drawings on paper, or using the computer program.

Multiple Means of Engagement
- The task connects students' general knowledge about geometric figures in a real-world context.
- The brainstorming for the task connects math with the students' own environment.
- Prior knowledge about congruence is activated as they discuss geometric figures.

Preparatory Activities

Suggested Materials
- Geogebra (Prior to this lesson, students should have some experience working with a dynamic geometry software program.)
- Initial file for the students to use. (File should contain two congruent triangles with SAS labeled (see Figure 1 below). Students should be able to move the original triangle ABC as well as change the measures of the sides and/or angles. This allows students to observe what their changes do to the second triangle and helps them understand the nature of congruence and the rigid motions needed to map one triangle onto the other.)

  (Figure 1)



- Student Worksheet
- Presentation of different congruent shapes: wallpaper patterns, quilts, bathroom tiles, kaleidoscopes, Ferris wheels, automobile manufacturing plants, stained glass windows, sidewalks, bridges, or playground equipment
- Extension

Extension Activities

Common Core State Standard

- Apply geometric concepts in modeling situations
    - CCSS.MATH.CONTENT.HSG.MG.A.3 Apply geometric methods to solve design problems (e.g., congruence and rigid motions to design an object or structure to satisfy physical constraints).

When students are ready, challenge them to create their own designs by applying congruence and rigid motions. Students research designs or structures that use congruent figures and create a poster that illustrates what they have learned. Provide students with the following instructions and questions:

Next we want to explore where congruence shows up in our daily lives.

- Do an Internet search and find a design or structure that uses congruent figures.
- Use rigid transformations to prove that parts of your chosen design or structure are, in fact, congruent.
- After you prove the existence of congruent figures, discuss with your group why congruence is an important component of the design or structure.
- Finally, make your own design or structure incorporating congruent figures in an important way. Create a poster of your design or structure. Include a picture of your design or structure, information about the use of congruent figures, and an argument for why the congruent figures were important components of your design or structure.

In the extension activity, students extend knowledge gained in previous activities to apply to a real-world setting. The previous methods of eliciting, interpreting, and acting on evidence should be used throughout this activity. Use the previous suggestions to scaffold the student understanding. This will help strengthen and deepen their understanding of rigid motions, congruence, and definitions. Students previously, in a pure mathematics activity, used justifying, communicating, and reasoning to prove two triangles were congruent. In this part of the lesson, students continue to use justifying, communicating, and reasoning to prove two triangles are congruent in a real-world setting. The students should continue to use a series of rigid motions and justify when there is more than one series of rigid motions that exist. Students should construct their arguments using the properties of rigid motions as justification in their real-world problems.