

Division Mistake Detectives

Fourth Graders Come to the Rescue!

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Oh man!!!

Ben just doesn't understand why two of his quotients are not correct. Can you help Ben?

$$\begin{array}{r} \underline{147} \\ 5 \overline{)785} \\ \underline{-5} \\ 28 \\ \underline{-25} \\ 35 \\ \underline{-35} \\ 0 \end{array} \checkmark$$

$$\begin{array}{r} \underline{507} \\ 4 \overline{)2528} \\ \underline{-25} \\ 028 \\ \underline{-28} \\ 0 \end{array} \checkmark$$

$$\begin{array}{r} \underline{811} \\ 8 \overline{)6488} \\ \underline{-64} \\ 08 \\ \underline{-8} \\ 0 \end{array}$$

How can we help Ben???

$$\begin{array}{r} \underline{147} \\ 5 \overline{)785} \\ \underline{-5} \\ 28 \\ \underline{-25} \\ 35 \\ \underline{-35} \\ 0 \end{array} \quad \checkmark$$

$$\begin{array}{r} \underline{507} \\ 4 \overline{)2528} \\ \underline{-25} \\ 028 \\ \underline{-28} \\ 0 \end{array} \quad \checkmark$$

$$\begin{array}{r} \underline{811} \\ 8 \overline{)6488} \\ \underline{-25} \\ 028 \\ \underline{-28} \\ 0 \end{array}$$

- What is his mistake?
- When does he make it?
- Why is one answer correct?
- How will you show him the correct way to divide?

Today we will

- Analyze error patterns that students may make when they are using the division algorithm without understanding it.
- Explain how error patterns can be corrected by using
 - Estimation
 - Repeated subtraction
 - Partial quotients
 - Concrete models or drawings
- Demonstrate the meaning of the division algorithm.

Mistake Detectives' Vocabulary

- Error Pattern: When a student makes the same mistake over and over again.
- Decomposing: Separating the values of numbers to make groups of tens, hundreds, thousands, etc. ($273 = 200 + 70 + 3$)
- Division Algorithm: The usual way of solving an division problem without using drawings or models.
- Division Problem:
$$\text{Dividend} \div \text{Divisor} = \text{Quotient} \text{ or } \text{Divisor})\text{Dividend}$$

Quotient
- Remainder: The amount left over after the dividend has been divided equally.

• Place Value:

Hundreds	Tens	Ones
3	2	6

Mistake Detectives' Vocabulary

$$\begin{array}{r} \underline{811 \text{ R } 1} \\ 8 \overline{)6489} \\ \underline{-64} \\ 08 \\ \underline{-8} \\ 09 \\ \underline{-8} \\ 1 \end{array}$$

Remainder:

The amount left over after the dividend has been divided equally.

This remainder can be interpreted as

$$6489 \div 8 = 811 \text{ with } 1 \text{ left over}$$

or

$$6489 = 811(8) + 1$$

Solve the following problems:

$$6 \overline{)789}$$

$$2 \overline{)684}$$

$$5 \overline{)5995}$$

Did you get the correct quotients?

$$\begin{array}{r} \underline{131 \text{ R } 3} \\ 6 \overline{)789} \\ \underline{-6} \\ 18 \\ \underline{-18} \\ 09 \\ \underline{-6} \\ 3 \end{array}$$

(131 with 3 left over)

$$\begin{array}{r} \underline{342} \\ 2 \overline{)684} \\ \underline{-6} \\ 08 \\ \underline{-8} \\ 04 \\ \underline{-4} \\ 0 \end{array}$$

$$\begin{array}{r} \underline{1199} \\ 5 \overline{)5995} \\ \underline{-5} \\ 09 \\ \underline{-5} \\ 49 \\ \underline{-45} \\ 45 \\ \underline{-45} \\ 0 \end{array}$$

Yikes !!!

Zoe solved the problems and got two of them wrong. What mistake did she make?

$$\begin{array}{r} 111 \\ \hline 6 \overline{)789} \end{array} \quad \checkmark$$

$$\begin{array}{r} 342 \\ \hline 2 \overline{)684} \end{array}$$

$$\begin{array}{r} 1111 \\ \hline 5 \overline{)5995} \end{array} \quad \checkmark$$

Step 1: Describe the error pattern.

$$\begin{array}{r} 111 \\ \hline 6 \overline{)789} \end{array} \quad \checkmark$$

$$\begin{array}{r} 342 \\ \hline 2 \overline{)684} \end{array}$$

$$\begin{array}{r} 1111 \\ \hline 5 \overline{)5995} \end{array} \quad \checkmark$$

- What is her mistake?
- When does she make it?
- Why is one answer correct?
- Do you see an error pattern?

Step 1: Describe the error pattern.

$$\begin{array}{r} 111 \\ \hline 6 \overline{)789} \end{array} \quad \checkmark$$

$$\begin{array}{r} 342 \\ \hline 2 \overline{)684} \end{array}$$

$$\begin{array}{r} 1111 \\ \hline 5 \overline{)5995} \end{array} \quad \checkmark$$

Zoe has totally forgotten about place value. She is dividing each digit of the dividend separately without subtracting and decomposing. The middle problem is correct because no decomposing was necessary.

Step 2: Help Zoe correct her error pattern.

$$\begin{array}{r} 111 \\ \hline 6 \overline{)789} \end{array} \checkmark$$

$$\begin{array}{r} 342 \\ \hline 2 \overline{)684} \end{array}$$

$$\begin{array}{r} 1111 \\ \hline 5 \overline{)5995} \end{array} \checkmark$$

- Estimation
- Repeated subtraction
- Partial quotients
- Concrete models or drawings

Estimation

$$\begin{array}{r} 111 \\ \hline 5 \overline{)975} \end{array}$$



$$2 \overline{)3698}$$

$$8 \overline{)8423}$$

$$3 \overline{)9275}$$

Estimation

975 is about 1,000.

$1,000 \div 5 \approx 200$.

My quotient will be about 200.

Algorithm

$$\begin{array}{r} 195 \\ \hline 5 \overline{)975} \\ -5 \\ \hline 47 \\ -45 \\ \hline 25 \\ -25 \\ \hline 0 \end{array}$$

Repeated Subtraction: $45 \div 9 = 5$

Repeated Subtraction

$$45 - 9 = 36$$

$$36 - 9 = 27$$

$$27 - 9 = 18$$

$$18 - 9 = 9$$

$$9 - 9 = 0$$

How many groups of 9 are there?
Are any ones left over?

$$9 \overline{)45}$$

$$\underline{-9}$$

$$36$$

$$\underline{-9}$$

$$27$$

$$\underline{-9}$$

$$18$$

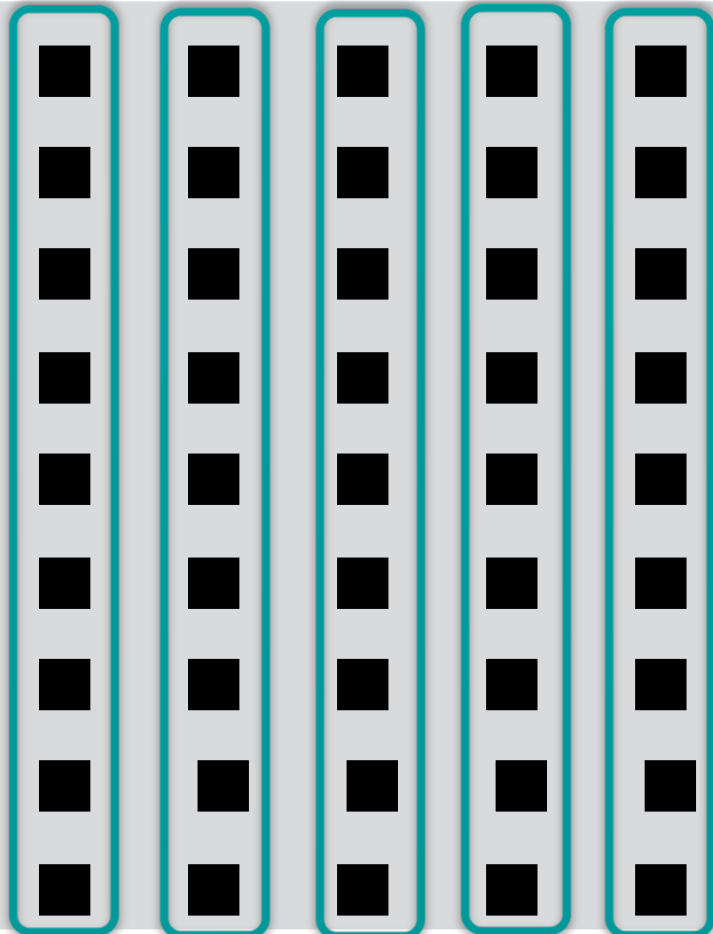
$$\underline{-9}$$

$$9$$

$$\underline{-9}$$

$$0$$

Concrete Model (Take away groups of 9)



Repeated Subtraction – your turn!

Quotient

Divisor)Dividend

- Write a division problem in your math journal.
- Solve by using repeated subtraction.
- Then solve by using the algorithm.
- When could you use repeated subtraction?
- When would another approach be better?

Partial Quotients

This method involves estimation!

OR

$$\begin{array}{r} 6 \overline{)789} \\ \underline{-600} \\ 189 \\ \underline{-120} \\ 69 \\ \underline{-60} \\ 9 \\ \underline{-6} \\ 3 \end{array} \quad \begin{array}{l} 100 \times 6 = 600 \\ 20 \times 6 = 120 \\ 10 \times 6 = 60 \\ + 1 \times 6 = 6 \\ \underline{131 \text{ with } 3} \\ \text{left over} \end{array}$$

$$\begin{array}{r} 6 \overline{)789} \\ \underline{-600} \\ 189 \\ \underline{-180} \\ 9 \\ \underline{-6} \\ 3 \end{array} \quad \begin{array}{l} 100 \times 6 = 600 \\ 30 \times 6 = 180 \\ + 1 \times 6 = 6 \\ \underline{131 \text{ with } 3} \\ \text{left over} \end{array}$$

OR??

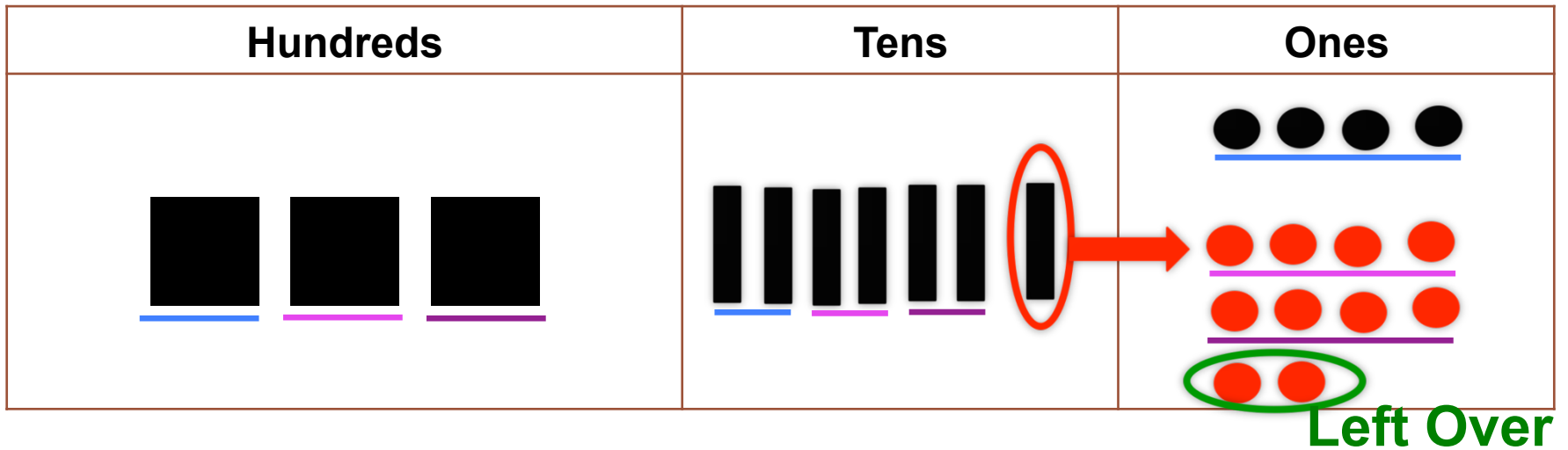
Partial Quotients – your turn!

$$5 \overline{)729}$$

1. Work with a partner. How many ways can you solve this problem using partial quotients?
2. Then solve the problem using the division algorithm.

Concrete Models or Drawings

$$374 \div 3 = 124 \text{ with } 2 \text{ left over}$$



Divide into 3 equal groups:



Your turn!

Choose a problem. Solve with concrete models or drawings and then by algorithm.

$$\begin{array}{r} 111 \\ 5 \overline{)575} \end{array}$$



$$2 \overline{)698}$$

$$4 \overline{)433}$$

$$3 \overline{)275}$$

Are there other ways to help Zoe?

$$\begin{array}{r} 111 \\ \hline 6 \overline{)789} \end{array} \quad \checkmark$$

$$\begin{array}{r} 342 \\ \hline 2 \overline{)684} \end{array}$$

$$\begin{array}{r} 1111 \\ \hline 5 \overline{)5995} \end{array} \quad \checkmark$$

- Estimation
- Repeated subtraction
- Partial quotients
- Concrete models or drawings
- Other ideas?

Explain two ways that you will use to help Zoe.

$$\begin{array}{r} 111 \\ \hline 6 \overline{)789} \end{array}$$



$$\begin{array}{r} 342 \\ \hline 2 \overline{)684} \end{array}$$

$$\begin{array}{r} 1111 \\ \hline 5 \overline{)5995} \end{array}$$



Zoe made so many mistakes because _____.
I will demonstrate the correct way by _____ and
_____. I chose these two methods because _____.
I will then show her how to divide using the
algorithm.

Solve the following problems:

$$3 \overline{)6067}$$

$$5 \overline{)7950}$$

$$2 \overline{)4862}$$

Did you get the correct quotients?

$$\begin{array}{r} 2022 \text{ } R \text{ } 1 \\ \hline 3 \overline{)6067} \\ \underline{-6} \\ 006 \\ \underline{-6} \\ 07 \\ \underline{-6} \\ 1 \end{array}$$

(2022 with 1 left over)

$$\begin{array}{r} 1590 \\ \hline 5 \overline{)7950} \\ \underline{-5} \\ 29 \\ \underline{-25} \\ 45 \\ \underline{-45} \\ 00 \end{array}$$

$$\begin{array}{r} 2431 \\ \hline 2 \overline{)4862} \\ \underline{-4} \\ 08 \\ \underline{-8} \\ 06 \\ \underline{-6} \\ 02 \\ \underline{-2} \\ 0 \end{array}$$

RATS!!!

Jack solved these problems and got two of them wrong. What is his error pattern?

$$\begin{array}{r} 222 \text{ R } 1 \quad \checkmark \\ \underline{3)6067} \\ -6 \\ \hline 006 \\ -6 \\ \hline 07 \\ -6 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 159 \quad \checkmark \\ \underline{5)7950} \\ -5 \\ \hline 29 \\ -25 \\ \hline 45 \\ -45 \\ \hline 00 \end{array}$$

$$\begin{array}{r} 2431 \\ \underline{2)4862} \\ -4 \\ \hline 08 \\ -8 \\ \hline 06 \\ -6 \\ \hline 02 \\ -2 \\ \hline 0 \end{array}$$

Step 1: Describe the error pattern.


$$\begin{array}{r} 222 R 1 \checkmark \\ \hline 3 \overline{)6067} \\ \underline{-6} \\ 006 \\ \underline{-6} \\ 07 \\ \underline{-6} \\ 1 \end{array}$$


$$\begin{array}{r} 159 \checkmark \\ \hline 5 \overline{)7950} \\ \underline{-5} \\ 29 \\ \underline{-25} \\ 45 \\ \underline{-45} \\ 00 \end{array}$$

$$\begin{array}{r} 2431 \\ \hline 2 \overline{)4862} \\ \underline{-4} \\ 08 \\ \underline{-8} \\ 06 \\ \underline{-6} \\ 02 \\ \underline{-2} \\ 0 \end{array}$$

- What is his mistake?
- When does he make it?
- Why is one answer correct?
- Do you see an error pattern?

Step 2: Explain two ways that you will use to help Jack.


$$\begin{array}{r} 222 \text{ R } 1 \\ \hline 3 \overline{)6067} \\ \underline{-6} \\ 006 \\ \underline{-6} \\ 07 \\ \underline{-6} \\ 1 \end{array}$$



$$\begin{array}{r} 159 \\ \hline 5 \overline{)7950} \\ \underline{-5} \\ 29 \\ \underline{-25} \\ 45 \\ \underline{-45} \\ 00 \end{array}$$


$$\begin{array}{r} 2431 \\ \hline 2 \overline{)4862} \\ \underline{-4} \\ 08 \\ \underline{-8} \\ 06 \\ \underline{-6} \\ 02 \\ \underline{-2} \\ 0 \end{array}$$

- Estimation
- Repeated subtraction
- Partial quotients
- Concrete models or drawings
- Other ideas?

Step 2: Explain two ways that you will use to help Jack.

$$\begin{array}{r}
 222 \text{ R } 1 \\
 \hline
 3 \overline{)6067} \\
 \underline{-6} \\
 006 \\
 \underline{-6} \\
 07 \\
 \underline{-6} \\
 1
 \end{array}$$


$$\begin{array}{r}
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 \end{array}$$


$$\begin{array}{r}
 2431 \\
 \hline
 2 \overline{)4862} \\
 \underline{-4} \\
 08 \\
 \underline{-8} \\
 06 \\
 \underline{-6} \\
 02 \\
 \underline{-2} \\
 0
 \end{array}$$

Jack made mistakes because _____. I will demonstrate the correct way by _____ and _____. I chose these two methods because _____. I will then show him _____.

What kinds of
division errors have
you made?



Be a Mistake Detective!

- Write five division problems with quotients that have an error pattern. Be sure to include one or two problems that have the correct answer.
- Describe the error pattern.
- Explain how you will help the student correct the mistake. Use writing, symbols, and pictures in your explanation.
- Write the five division problems again using the correct algorithm and with the correct quotients.
- Share your work.



Mistake Detective Rubric

4	3	2	1
<p>Fully accomplishes the purposes of the task:</p> <ul style="list-style-type: none">• Shows full grasp and use of the central mathematical ideas.• Recorded work communicates thinking clearly using some combination of written, symbolic, or visual means.• Computational work using the algorithm is correct.	<p>Substantially accomplishes the purposes of the task:</p> <ul style="list-style-type: none">• Shows essential grasp of the central mathematical ideas.• Recorded work in large part communicates thinking using some combination of written, symbolic, or visual means.• Computational work using the algorithm is correct.	<p>Partially accomplishes the purposes of the task:</p> <ul style="list-style-type: none">• Shows partial but limited grasp of the central mathematical ideas.• Recorded work may be incomplete, misdirected, or not clearly presented• There are mistakes in the computational work that uses the algorithm.	<p>Little or no progress toward accomplishing the purposes of the task:</p> <ul style="list-style-type: none">• Shows little or no grasp of the central mathematical ideas.• Recorded work is barely (if at all) comprehensible.• There are mistakes in the computational work that uses the algorithm.